MEKELLE UNIVERSITY

COLLEGE OF NATURAL AND COMPUTATIONAL SCIENCES

DEPARTMENT OF MATHEMATICS

A

THESIS

ON

Mathematical Modeling of Fish Population Dynamics With Harvesting In Tekeze Dam

By

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A

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Mathematical Modeling of Fish Population Dynamics With Harvesting In Tekeze Dam

Submitted to the Department of Mathematics in Partial Fulfillment of the Requirements for the Degree of Master of Science in Mathematics.

By

Gebrekires Hadush

Under the supervision of

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Co-advisor: Yohannes Yirga (PhD)

APPROVAL SHEET

Mekelle University

College of Natural and Computational Sciences
Department of Mathematics

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Declaration

I, Gebrekires Hadush, declare that this thesis entitled "Mathematical Modeling of Fish Population Dynamics With Harvesting in Tekeze dam" is my original work and has not been presented for any other award, and that all sources of materials used in this thesis are duly acknowledged.I confirm that: This work was done wholly or mainly while in candidature for a research degree at this University.

Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated. Where I have consulted the published work of others, this is always clearly attributed, and I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is my own work.

I have acknowledged all main sources of help. Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Name of candidate: Gebrekires Hadush

Signed:

Date: 05/02/2026

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Gebrekires Hadush
December, 2025

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Abbreviations

N	**p**oppulation **S**ize
K	**C**arrying **C**apacity
r	**I**ntrinsic **G**rowth **R**ate
t	**T**ime **S**pan
H	**H**arvesting **R**ate
Sin	**S**in **F**unction
MSY	Maximum Sustainable Yield
CPUE	Catchability Per Unit Effort
E	Fishing Effort
FAO	Food and Agriculture Organization
q	Catchability coefficient

Dedication

I want to dedicate this work to Tsehaytu G/her, for her love and encouragement during my growth. She was my grand mother and I attended my education with the help of my grand mother until I join university.
Tsehaytu G/her passed away in early 2012 when I was at university and we all her sons regret that she will not be able to see the results of her kind efforts.

Abstract

This study analyzes the mathematical models to determine sustainable harvesting strategies for Tilapia fish population in Tekeze dam, Ethiopia. Even though Tilapia fish farming has been locally commercialized, the use of mathematical models in determining harvesting strategies has not been widely applied in Tekeze dam. Logistic model is appropriate for population growth of fishes when overcrowding, over fishing and competition for resource are taken into consideration. Using a logistic growth models, it formulates and compare three harvesting strategies: constant, periodic and proportional. The carrying capacity of the dam was first estimated at 153312 tonnes using a phosphorus budget model. Analysis reveals a Maximum Sustainable Yield (MSY) of 30662 tonnes per year under constant harvesting, representing a critical threshold beyond which the population collapses. Proportional harvesting demonstrates high sensitivity to fishing effort, with a clear extinction threshold. In contrast, periodic harvesting emerges as the most sustainable and resilient strategy. By incorporating recovery periods, it maintains ecological balance, optimizes long term yield, and presents stock depletion.

Numerical simulation confirm the optimal harvesting rate converges to half the intrinsic growth rate(h=r/2) for yield maximization. The findings provide a scientific basis for management, recommending the adoption of regulated periodic closures, effort limits, and community-led monitoring. This work offers a practical, model-driven framework to transform the Tekeze Dam fishery in to a sustainable resources, balancing food security with ecosystem health.

These findings can assist fish harvesters neighboring to Tekeze dam, to increase fish supply to meet the demand for Tilapia fish.

Keywords: Mathematical modeling, Fish population dynamics, Logistic model, Harvesting strategies, Maximum Sustainable Yield, Tekeze Dam, Tilapia Fish, Sustainable fisheries.

Chapter 1

Introduction

1.1 Background of the study

Globally, fish is an important commodity contributing in nutrition, food security and employment in the world. There are many types of fish that live in every region of the world. There are about 40,000 fish species in the world. According to Food and Agriculture Organization (FAO) [22], fishery is an important sector used for food security and source of cheap protein for the poor living in developing countries. FAO reports that nearly 90% of global marine fish stoks are fully exploited or over-fished. Fisheries have the most significant contribution in economy of developing countries where means of livelihoods are very limited.

In Afria, Inland fisheries play a crucial role in food security and poverty alleviation, specially in regions wih limited alternative protien sources. In Ethiopia, about half a million people depend on capture fishery directly and indirectly as means of livelihoods (Tesfaye and Wolff, 2014)[23]. The country has many lakes and reservoirs, small water bodies and large floodplain areas distributed all over the country from lowland to highlands covering a total surface area of about 13,637 km2 (Tesfaye and Wolff, 2014) [23]. The fish diversity of the country is estimated to over 200 species (Tesfaye and Wolff, 2014; Getahun, 2017) [23]. In the study area, Tekeze dam Tsegay Teame [18] recorded 11 fish species belonging to 8 genera, 4 amilies and 3 orders.

Fishes are important in different ways. They can be used as a source of food, money, for human being as well as the source of food for the other aquatic animals.

Fish harvesting is the process of gathering the fish by using the equipment. Fishes are harvested for their highly nutritious meat and for the oil that is extracted and used as a food product or as an ingredient for a wide variety of commercially prepared products. Mathematical models have been used widely to estimate the population dynamics of animals for so many years as well as the human population dynamics. In recent years, the use of mathematical models has been extended to agriculture sector especially in cattle farming to ensure continuous and optimum supply.

The logistic growth model in terms of harvesting has been used to study the fish farming. The most important for successful management of harvested populations is that harvesting strategies are sustainable, not leading to instabilities or extinctions and produces great results for the year with little variation between the years. Therefore, it can supply the market demand throughout the year.

Population dynamics refers to changes in the sizes of populations of organisms through time.

Maximum sustainable yield (MSY) is theoretically, the largest yield (or catch) that can be taken from a species' stock over an indefinite period. Sustainability of renewable resource means exploiting natural resources without destroying the ecological balance. Tilapia fish farming has been an important source in some areas of the world and it is well suited for farming since they are fast growing and hardy. Tilapia fish also can establish strong population in very short time duration if the environment is right [19]. This has made tilapia fish a very important protein source. The period of maturity for the tilapia fish is 6 months and estimates that 80% will survive to maturity [20].

The fish production estimation from 106 waterbodies in Ethiopia was estimated to be 94,500 tons per year (73,100 tons/year from lentic and 21,400 tons/year from lotic systems) (Tesfaye and Wolff, 2014) [23]. However, the actual fish production in Ethiopia is far below the potential and the production status is uneven across water bodies. In some lakes such as Tana, Chamo, Abaya, Hayq, Ziway and Tekeze dam, fishing is beyond the maximum sustainable production potential and the problem of overfishing has been already reported in these water bodies (Tesfaye and Wolff, 2016) [24]. However, in some lakes which are located in remote areas such as Lakes Maybar and Golbo the fishery is unexploited and further development is required (Tessema and Geleta, 2013; Tesfaye and Wolff, 2014; Lakew et al., 2016)[23]. The most important factors limiting fish production are the use of traditional fishing methods, food habits of the people, poor facilities along the fish value chains, poor fishery regulation implementation system, pollution, weak coordination among water sectors and generally neglectence to the

fishery sector (Abera, 2017; Abebe and Chalchisa, 2019) [25]. Lake Hayq is one of the highland lakes of Ethiopia with significant fishery activities and source of livelihoods directly and indirectly for the people living around the lake. About a decade ago, fish production of Lake Hayq was very high and was able to support about 2000 fishermen and fish traders (Fetahi et al., 2011a; Tessema and Geleta, 2013; Seid, 2016) [26]. However, the fish production trend especially of Nile tilapia has been reduced from time to time (Worie, 2009; Tessema and Geleta, 2013; Worie and Getahun, 2014; Mekonen et al., 2019) 27. Though many studies were conducted on physical limnology (Kebede et al., 1992), phytoplankton and zooplankton community structures and energy flows and food web structures (Fetahi et al., 2011a, 2011b, 2014), lake morphometric and land use and land cover (Mohammed et al., 2013, 2015), water quality (Ruchi et al., 2016) [28], there is limited information on socioeconomic importance of fisheries, status of the current fishing activities and its impact on Tilapia fish in Tekeze dam.
Therefore, the researcher of this study on Tilapia was aimed to model and analyze the fish population dynamics with various harvesting strategies in the case of Tekeze dam.

1.2 Statement of the problem

Tekeze dam is a vital aquatic ecosystem providing significant economic and nutritional benefits to the surrounding communities through its fishery resources. To ensure the long-term viability of these resources, fish population must be managed at levels that allow for both ecological stability and sustainable economic yield.
however, the fishery at Tekeze dam currently faces critical challenges that threaten its sustainability. Despite its potential, the dam plagued by unregulated over-fishing poses a significant threat to aquatic ecosystem leading to decline in fish populations and biodiversity. It compounded by the prevalence of traditional harvesting methods, which often utilize non-selective gear that disrupts the age struture of the fish before they reach reproductive maturity. This research addresses the critical need for mathematical models that can accurately simulate fish population dynamics under various harvesting scenarios. By understanding the implications of different harvesting strategies, this study aims to inform sustainable fisheries management practices that balance ecological integrity with economic viability.

Therefore, this research work addresses the following basic research questions to solve the problem in the dynamics of fish population with harvesting

- What are the long-term population dynamics of fish in a controlled environment without harvesting?

- How constant harvesting affect the stability and sustainability of fish population over time?

- How does periodic harvesting influences the resilience of fish population to environmental changes?

- How do proportional harvesting impact the structure and genetic diversity of fish population?

- What is the optimal harvesting rate that maximizes yield without leading to population decline?

1.3 Objective of the study

1.3.1 The main objective of the study

The general objective of this study was to analyze mathematical models of fish population dynamics that incorporates various harvesting strategies in Tekeze Dam.

1.3.2 The specific objectives of the study include

:

- To formulate mathematical model for Tilapia population without harvesting to establish a baseline for comparison.
- To perform stability and bifurcation analysis for each harvesting strategy and identify critical thresholds for sustainable harvesting.
- To investigate the harvesting strategies: constant, periodic and proportional harvesting, examining intervals on population dynamics.
- To compare the long-term outcomes of each harvesting strategy, providing recommendations for effective fishing management.
- To evaluate the optimal harvesting rate that maximizes yield of Tilapia fish population.

1.4 Significance of the study

The significant of this study are: To deliver evidence-based insights to inform regional and national fishery policies promoting sustainable resources use. Besides, it creates awareness of the society who are around the Dam and provide initial for further study on the effect of fish population dynamics. Finally, it identifies a Maximum Sustainable Yield (MSY) of 30662 tonnes/year and establishes periodic harvesting as the most sustainable strategy in Tekeze Dam.

1.5 Scope of the Study

The scope of this study is delimited to the mathematical modeling of fish population dynamics with harvesting in case of Tekeze dam in 2025. Analysis is based on logistic growth models with three harvesting strategies.

1.6 Limitation of the Study

This study was employed in Tekeze dam. Therefore, the researcher strongly agrees that the involvement of additional dams and lakes may help to gain more relevant and wider information. Moreover, since the research was new in the area that the study took place the researcher may encounter limitations of related research works in the title and lack of related reference books studied in Ethiopia and enforced mostly to use foreign sources.

Chapter 2

Review of related literature

2.1 Introduction: The Role of Modeling in Population Dynamics

Population dynamics refers to changes in the sizes of populations of organisms through time. Fish harvesting is the process of gathering the fish by using the equipment. Fishes are harvested for their highly nutritious meat and for the oil that is extracted and used as a food product or as an ingredient for a wide variety of commercially prepared products(FAO) [22].

Mathematical models have been used widely to estimate the population dynamics of animals for so many years as well as the human population dynamics. In recent years, the use of mathematical models has been extended to agriculture sector especially in cattle farming to ensure continuous and optimum supply.

The logistic growth model in terms of harvesting has been used to study the fish farming. The most important for successful management of harvested populations is that harvesting strategies are sustainable, not leading to instabilities or extinctions and produces great results for the year with little variation between the years. Therefore, it can supply the market demand throughout the year.

Maximum sustainable yield (MSY) is theoretically, the largest yield (or catch) that can be taken from a species' stock over an indefinite period. Sustainability of renewable resource means exploiting natural resources without destroying the ecological balance. Tilapia fish farming has been an important source in some areas of the world and it is well suited for farming since they are fast growing and hardy. Tilapia fish also can establish strong population in very short time duration if the environment is right[19]. Population Dynamics Populations grow in size when the birth rate exceeds the death rate. Thomas Malthus, in An Essay on the Principle of Population(1798), used un checked population growth to famously predict a global famine unless governments regulated family size—an idea later echoed by Mainland China's one-child policy. The reading of Malthus is said by Charles Darwin in his autobiography to have inspired his discovery of what is now the cornerstone of modern biology: the principle of evolution by natural selection.

2.2 Foundation of Population Growth Models

2.2.1 The Mathusian Exponential Model

The earlier formal population model, proposed by Thomas Malthus in 1798, describes unbounded exponential growth: $dN/dt = rN$, where r is the intrinsic growth rate. While foundation, this model is biologically unrealistic over the long term, as it ignores environmentals limitations on resources.

2.2.2 The Logistic Growth Model

The Malthusian growth model is the grandfather of all population models, and we begin this study with a simple derivation of the famous exponential growth law. Unchecked exponential growth obviously does not occur in nature, and population growth rates may be regulated by limited food or other environmental resources, and by competition among individuals within a species or across species. We will develop model and The model is the well known logistic equation. The exponential growth law for population size is unrealistic over long times. Eventually, growth will be checked by the over-consumption of resources. We assume that the environment has an intrinsic carrying capacity K, and populations larger than this size experience heightened death rates. To model population growth with an environmental carrying capacity K, we look for a nonlinear equation of the form

$$\frac{dN}{dt} = rNF(N) \tag{2.1}$$

where F(N) provides a model for environmental regulation. This function should satisfy $F(0) = 1$ (the population grows exponentially with growth rate r when N is small), $F(k) = 0$ (the population stops growing at the carrying capacity), and $F(k) < 0$ when $N > K$ (the population decays when it is larger than the carrying capacity). The simplest function $F(N)$ satisfying these conditions is linear and given by $F(N) = 1 - \frac{N}{K}$. The resulting model is the well-known logistic equation,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{2.2}$$

Because a these mathematical models are nonlinear differential equations, mathematical methods to analyze such equations will be developed[21].

2.3 Harvesting Strategy in Fishery Models

The Rahmani M.H. Doust[2] paper deals with the logistic growth model having harvesting factor, which is studied in two cases constant and non-constant. In fact, the nature of equilibrium points and solutions behavior has been analyzed for both of the constant and non-constant cases by finding the first integral, solution curve and phase diagram. Finally, a theorem describing the stability of a real model of single species is proved. From R.M.H. Doust study if the harvesting factor is constant, then the equilibrium point is stable and if the harvesting factor is non constant, then the equilibrium point is unstable.

Lev V. Idel and Mei Wang (2008)[1] developed a new fishing effort model which relies on the density effect of fish population.

They obtained new differential equations to describe certain standard fisheries management strategies.

This study concludes that a control parameter β (the magnitude of the effect of fish population size on the fishing effort function), changes not only the rate at which the population goes to equilibrium, but also the equilibrium values.

They used different strategies to develop a new fishing effort model by comparing the strategies. (i) In the case of proportional harvesting, increase in β will stabilize the population faster. (ii) In proportional threshold harvesting, increase in β will converge

$N(t)$ to stable equilibrium point faster $N(0)$ is above the equilibrium solution. However, if $N(0)$ is below the equilibrium solution, increase in β will cause $N(t)$ to converge to stable equilibrium point slower. (iii) In seasonal harvesting strategy due to the periodic nature of λ (proportional harvesting rate) it tends to have higher stable equilibrium point than proportional harvesting strategy, i.e. changes the value of stable equilibrium point.

According to Ratneesh Suri (2008) [6], the purpose of fisheries management is to achieve the sustainable development of the activity, so that the future generation can also benefit from the resource. The optimal harvesting strategy usually maximizes an economically important objective function formed by the harvester which can lead to the extinction of the resource population.

Therefore, sustainability has been far more difficult to achieve than is commonly thought; fish populations are becoming increasingly limited and catches are declining due to overexploitation. In his research he developed optimal harvesting strategy and formulated to maximize the net profit from harvesting.

According to Mohamed Faris Laham, et.al (2012) [7] two logistic growth models have been used namely constant harvesting and periodic harvesting. Logistic growth model is appropriate for population growth of animal when overcrowding and competition resources are taken into consideration [7]. The objectives of their study were to estimate the highest continuing yield from fish harvesting strategies implemented. Secondly, the study predicted the optimum quantity for harvesting that can ensure the tilapia fish supply is continuous. Finally, to compare the results obtained between the two strategies, the best harvesting strategy for the selected fish farm is periodic harvesting. Periodic harvesting strategy can be used to improve productivity, shorten investment return time. By using the constant harvesting the fish farming does not have enough time to recover the fish population. According to Corinne Wentworth, et.al (2011) [9] fishery management is the consideration of the ecological effects of harvesting.

Fisherman work to provide fish for a growing human population but because of this some fish populations have been dangerously declining.

It is important to balance ecological and economic needs. Various deterministic models of fishery populations were investigated.

A simple logistic model, a skewed logistic model with a quadratic term, and a model that demonstrates the Allee effect has all been considered with a constant harvest rate as well as time dependent harvesting.

Optimization and numerical calculations were used to determine the harvest rate that produces maximum yield under different population density scenarios.

Neelima Daga, et.al. (2014)[9] have considered a prey-predator fishery model with prey dispersal in a two-patch environment, one is assumed to be a free fishing zone and the other is a reserved zone, where fishing and other extractive activities are prohibited. The local and global stability analysis has been carried out.

Biological equilibria of the system along with the conditions of their existence are obtained. Criteria for the coexistence of predator-prey system are obtained. Numerical simulation has also been performed in support of the analytic result.

They obtained that whether in the absence or presence of predator, the fishing populations may be sustained at an appropriate equilibrium level.

B. Dubeya, et.al. (2002) [8] developed a mathematical model to study the dynamics of a fishery resource system in an aquatic environment.

Biological and bionomic equilibria of the system are obtained, and criteria for local stability, instability and global stability of the system were derived.

It was shown that even if fishery was exploited continuously in the unreserved zone, fish populations can be maintained at an appropriate equilibrium level in the habitat.

An optimal harvesting policy is also discussed using the Pontryagin's Maximum Principle. Under continuous harvesting of fish species outside the reserved zone, fish population may be maintained at an appropriate equilibrium level.

Optimal harvesting policy has been discussed and concluded that high interest will cause high inflation rate.

B Dubay [12] has developed a mathematical model to study the role of a reserved zone on the dynamics of prey-predator system and also established that the reserved zone has a stabilizing effect on predator-prey interactions.

Mellachervu Naga Srinivas, et.al. (2011)[15][3] propose and analyze a mathematical model to study the dynamics of a fishery resource system with stage structure in an aquatic environment that consists of two zones namely unreserved zone (fishing permitted) and reserved zone (fishing is strictly prohibited). In this model they introduce a stage structure in which predators are split into two kinds as immature predators and mature predators. It is assumed that immature predators cannot catch the prey and their foods are given by their parents (mature predators). It is also assumed that the fishing of immature predators prohibited in the unreserved zone and predator species are not allowed to enter inside the reserved zone. The local and global stability analysis has been specified. Biological and bionomical equilibrium points of the system were derived. Mathematical formulation of the optimal harvesting policy was given and its solution was derived in the equilibrium case by using Pontryagin's maximum principle. The vital role of reserved zone in aquatic environment for protection of fishery resources from its overexploitation is discussed by several researchers.

Kar and Matsuda [5] have studied the use of marine protect area on both biological and economical aspect. They investigated the impacts of the creation of Marine Protected Areas (MPAs), in both economic and biological perspectives. The economic indicator was assumed to be optimally managed. The biological indicator was taken as the stock density of the source. The basic fishery model was serving as the benchmark in comparing results with those that are derived from a model of two patchy populations. A crucial characteristic is the migration coefficient which describes biological linkages between protected and unprotected areas. Both economic and biological criteria are enhanced, after introducing a marine protected area, was presented.

Kar and Chakraborty [4] have developed a model to examine the effects of marine reserves on equilibrium levels of fish biomass, catch, predation and rent in a fishery. They have also considered the optimal area of the reserve and exploitation rate in the fishery.

Chakraborty and Kar [3] have studied a bio-economic model of a prey-predator fishery with protected area and also discussed the system numerically and observed that marine protect area can be used as an effective management tool to improve resource rent under a number of circumstances.

Chapter 3

Methods and Model Formulation

3.1 Materials and methods:Description of the study Area Tekeze Dam

Tekeze reservoir is a hydropower reservoir constructed on 2009 over the Tekeze River. Tekeze River is a major river in Ethiopia and it is a Nile tributary.

Tekeze dam is with a maximum length of 75 km and maximum width of 6 km, and covering an area of about 160.4km^2. According to National Statistics of Agency (NSA) (2008) Tekeze River is 608 kilometers long.

Mana, Tsilare, Seletsa, Avera and Ariqua rivers are the main tributaries of the Tekeze River joined in to the reservoir. The canyon which it has created is the deepest in Africa and one of the deepest in the world, at some points having a depth of over 2000 meters. Tekeze River originates in the central Ethiopian Highlands near Mount Qachen within Lasta, at $14^0 11N37^0 31.7E$ and $14.18.3^0 N37.5283^0 E$. The reservoir is located at an elevation of 1107m above sea level. It is approximately 155 km from Mekelle city. The capacity of the reservoir is about 9.293 billion m^3 of water. Although Tekeze reservoir is constructed entirely in Tigray region, the water of the reservoir is shared by Amhara region. The reservoir is communal for five districts (Tanqua Abergelle district from Tigray region and Abergelle, Zikuala, Sahla and Tselemti districts from Amhara region). The main aim of constructing of the Reservoir was to produce electricity, but the reservoir fisheries were later recognized as a significant socio-economic importance to Tigray and Amhara people. The reservoir also facilitates the transportation of goods and passengers and the provision of services for Tigray and Amhara people found on both sides of the reservoir[18].

Tilapia fish farming has been an important source in some areas of the world and it is well suited for farming since they are fast growing and hardy. Tilapia fish also can establish strong population in very short time duration if the environment is right[19]. This has made tilapia fish a very important protein source. The period of maturity for the tilapia fish is 6 months and estimates that 80% will survive to maturity [20].

The methods we were employed to address the problems in the dynamics of fish population with harvesting is that we will develop a mathematical model that mimics the dynamics using a system of nonlinear differential equations.

FIGURE 3.1: The Tilapia Fish In Tekeze dam

The analysis of the model was done using stability of the equilibrium points of the system of equations respectively. Usually, it is difficult to find the analytical solutions for non-linear differential equations. Thus, we use numerical solution of the fish harvesting models were done using MATLAB command ode45 to solve all the differential equations. Detail stability analysis of each logistic growth models are incorporated.

3.2 The Logistic Models with Various Harvesting Strategies

We consider the logistic growth equation to model a fish population in the absence of fishing.

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{3.1}$$

where N is the size of the fish population, r is the intrinsic growth rate due to reproduction and K is the carrying capacity of the environment.

3.2.1 The logistic growth model With constant harvesting

The model modifies the logistic equation to include a constant harvesting rate H:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H \tag{3.2}$$

where N is the size of the fish population, r is the intrinsic growth rate due to reproduction and K is the carrying capacity of the environment.

3.2.2 The logistic growth model With periodic harvesting

This model introduces periodic harvesting at intervals t:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - h\left(1 + sin(2\pi t)\right) \tag{3.3}$$

where N is the size of the fish population, r is the intrinsic growth rate due to reproduction and K is the carrying capacity of the reservoir, m is , q is catchability coefficient , and E is fishing effort.

3.2.3 The logistic growth model With proportional harvesting

In scenario, the harvesting rate is proportional to the population size:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - mqEN \tag{3.4}$$

The methods we were employed to address the problems in the dynamics of fish population with harvesting is that we will develop a mathematical model that mimics the dynamics using a system of nonlinear differential equations. The analysis of the model will be done using stability of the equilibrium points of the system of equations respectively. Usually, it is difficult to find the analytical solutions for non-linear differential equations. Thus, we use numerical solution of the fish harvesting models were done using MATLAB command ode45 to solve all the differential equations. Detail stability analysis of each logistic growth models are incorporated.

3.3 MODEL FORMULATION

3.3.1 The Logistic equation

The exponential growth law for population size is unrealistic over long times. Eventually, growth will be checked by the over-consumption of resources. We assume that the environment has an intrinsic carrying capacity K, and populations larger than this size experience heightened death rates. To model population growth with an environmental carrying capacity K, we look for a nonlinear equation of the form

$$\frac{dN}{dt} = rNF(N) \tag{3.5}$$

where F(N) provides a model for environmental regulation. This function should satisfy $F(0) = 1$ (the population grows exponentially with growth rate r when N is small), $F(K) = 0$ (the population stops growing at the carrying capacity), and $F(k) < 0$ when $N > K$ (the population decays when it is larger than the carrying capacity). The simplest function $F(N)$ satisfying these conditions is linear and given by $F(N) = 1 - \frac{N}{K}$. The resulting model is the well-known logistic equation,

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{3.6}$$

An important model for many processes besides bounded population growth.

Although (3.6) is a nonlinear equation, an analytical solution can be found by separating the variables. Before we embark on this algebra, we first illustrate some basic concepts used in analyzing nonlinear differential equations. Fixed points, also called equilibrium, of a differential equation such as (3.6) are defined as the values of N where $\frac{dN}{dt} = 0$. Here, we see that the fixed points of (3.6) are $N = 0$ & $N = K$. If the initial value of N is at one of these fixed points, then N will remain fixed there for all time. Fixed points, however, can be stable or unstable. A Fixed point is stable if a small perturbation from the Fixed point decays to zero so that the solution returns to the Fixed point. Likewise, a Fixed point is unstable if a small perturbation grows exponentially so that the solution moves away from the Fixed point. Calculation of stability by means of small perturbations is called linear stability analysis. For example, consider the general one-dimensional differential equation using the notation $x^{'} = \frac{dx}{dt}$

$$x^{'} = f(x), \tag{3.7}$$

with $x^{'}$ a fixed point of the equation, that is $f(x_*) = 0$. To determine analytically if x_* is a stable or unstable fixed point, we perturb the solution. Let us write our solution $x = x(t)$ in the form

$$x(t) = x_* + \epsilon(t) \tag{3.8}$$

where initially (0) is small but different from zero. Substituting (3.8) into (3.7), we obtain $\epsilon^{'} = f(x_* + \epsilon) = f(x_*) + \epsilon f^{'}(x_*) + \cdots = \epsilon f^{'}(x_*) + \cdots$ where the second equality uses a Taylor series expansion of $f(x)$ about x_* and the third equality uses $f(x_*) = 0$. If $f^{'}(x_*) 0$, we can neglect higher-order terms in for small times, and integrating we have
$\epsilon(t) = \epsilon_0 e^{f^{'}(x_*)t}$
The perturbation (t) to the fixed point (x_*) goes to zero as $t \to \infty$ provided $f'(x_*) < 0$. Therefore, the stability condition on

$$x_* is \begin{cases} \text{stable fixed point if } f^{'}(x_*) < 0 \\ \text{unstable fixed point if } f^{'}(x_*) > 0 \end{cases}$$

Output

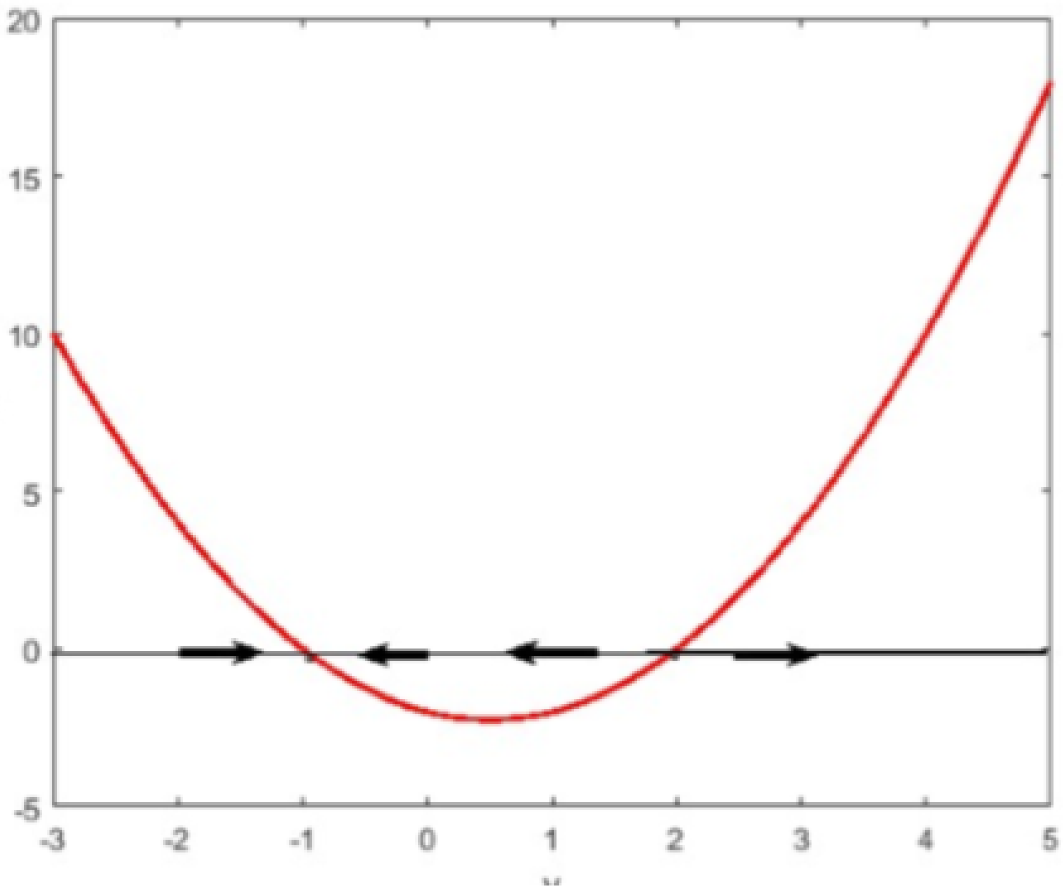

FIGURE 3.2: Stable and Unstable Fixed Points of Logistic Growth Model

Another equivalent but sometimes simpler approach to analyzing the stability of the fixed points of a one-dimensional nonlinear equation to plot $f(x)$ versus x.

The fixed points are the x-intercepts of the graph. Directional arrows on the x-axis can be drawn based on the sign of $f(x)$. If $f(x) < 0$, then the arrow points to the left; if $f(x) > 0$, then the arrow points to the right. The arrows show the direction of motion for a particle at position x satisfying

$x' = f(x)$. Fixed points with arrows on both sides pointing in are stable, and fixed points with arrows on both sides pointing out are unstable. In the logistic equation , the fixed points are $N_* = 0, K$. A sketch of

$$F(N) = rN\left(1 - \frac{N}{K}\right)$$

versus N, with $r, K > 0$ is immediately shows that $N_* = 0$ is an unstable fixed point and $N_* = K$ is a stable fixed point. The analytical approach computes $F^{'}(N) = r(1 - 2 \times \frac{N}{K})$, so that $FF^{'}(0) = r > 0$ and $F^{'}(K) = -r < 0$. Again we conclude that $N_* = 0$ is unstable and$N_* = K$ is stable. We now solve the logistic equation analytically. Although this relatively simple equation can be solved as is, we first non-dimensional to illustrate this very important technique that will later prove to be most useful. Perhaps here one can guess the appropriate unit of time to be 1/r and the appropriate unit of population size to be K. However, we prefer to demonstrate a more general technique that may be usefully applied to equations for which the appropriate dimensionless variables are difficult to guess. We begin by non-dimensional time and population size: $\tau = \frac{t}{t_*}$ and $\eta = \frac{N}{N_*}$ where t_* and N_* are unknown dimensional units. The derivative N is computed as $\frac{dN}{dt} = \frac{dN_*\eta}{d\tau}\frac{d\tau}{dt} = \frac{N_*d\eta}{t*d\tau}$ Therefore, the logistic equation becomes
$\frac{d\eta}{d\tau} = rt_*\eta\left(1 - \frac{N_*\eta}{K}\right)$
which assumes the simplest form with the choices $t_* = \frac{1}{r}$ and $N_* = K$.
Therefore, our dimensionless variables are $\tau = rt$,& $\eta = \frac{N}{K}$ and the logistic equation, in dimensionless form, becomes $\frac{d\eta}{d\tau} = \eta(1-\eta)$, with the dimensionless initial condition $\eta(0) = \eta_0 = \frac{N_0}{K}$, where N_0 is the initial population size. Note that the dimensionless logistic equation has no free parameters, while the dimensional form of the equation contains r and K. Reduction in the number of free parameters here, two r and K by the number of independent units here, also two time and population size is a general feature of nondenominational. The theoretical result is known as the Buckingham π Theorem. Reducing the number of free parameters in a problem to the absolute minimum is especially important before proceeding to a numerical solution. The parameter space that must be explored may be substantially reduced. Solving the dimensionless logistic equation can proceed by separating the variables. Separating and integrating from $\tau = 0$ to τ and η_0 to η yields
$\frac{d\eta}{\eta(1-\eta)} = d\tau$ The integral on the left-hand-side can be performed using the method of partial fractions: $\frac{1}{\eta(1-\eta)} = \frac{A}{\eta} + \frac{B}{(1-\eta)} = A(1-\eta) + B\eta$ and by equating the coefficients of the numerators proportional to η_0 and η_1, we find that A = 1 and B = 1. Therefore,

$\frac{d\eta}{\eta(1-\eta)} = \frac{d\eta}{\eta} + \frac{d\eta}{1-\eta)} = \ln\frac{\eta}{\eta_0} - \ln\frac{(1-\eta)}{(1-\eta_0)} = \ln\frac{\eta(1-\eta_0)}{\eta_0(1-\eta)} = \tau$ (from $\eta_0 to \eta$)
Solving for η, we first exponentiate both sides and then isolate η: $\frac{\eta(1-\eta_0)}{\eta_0(1-\eta)} = e^{\tau}$ or $\eta(1-\eta_0) = \eta_0 e^{\tau} - \eta\eta_0 e^{\tau}$ or $\eta(1-\eta_0+\eta_0 e^{\tau}) = \eta_0 e^{\tau}$, or $\eta = \frac{\eta_0}{\eta_0(1-\eta_0)e^{-\tau}}$ Returning to the dimensional variables, we finally have
$N(t) = \frac{N_0}{\frac{N_0}{K} + \left(\frac{1-N_0}{K}\right)e^{-rt}}$
There are several ways to write the final result. The presentation of a mathematical result requires a good aesthetic sense and is an important element of mathematical technique. When deciding how to write (6), I considered if it was easy to observe the following limiting results:
$N(0) = N_0; \lim_{t\to\infty N(t)} = K;$
$\lim_{K\to\infty N(t)} = N_0 \exp(rt)$

MATLAB code for FIGURE(3.3) $eta0 = [0.02, 0.2, 0.5, 0.8, 1, 1.2];$

$tau = linspace(0, 8, 1000);$

figure('Position',[100,100,800,600]); hold on; grid on; box on;

$colors = lines(length(eta0)); for i = 1 : length(eta0)\ eta = eta0(i)./(eta0(i) + (1 - eta0(i)).* exp(-tau));$

plot(tau, eta,'LineWidth',2,'Color',colors(i,:),... 'DisplayName',['η_0 =', $num2str(eta0(i))$]);
end

Output

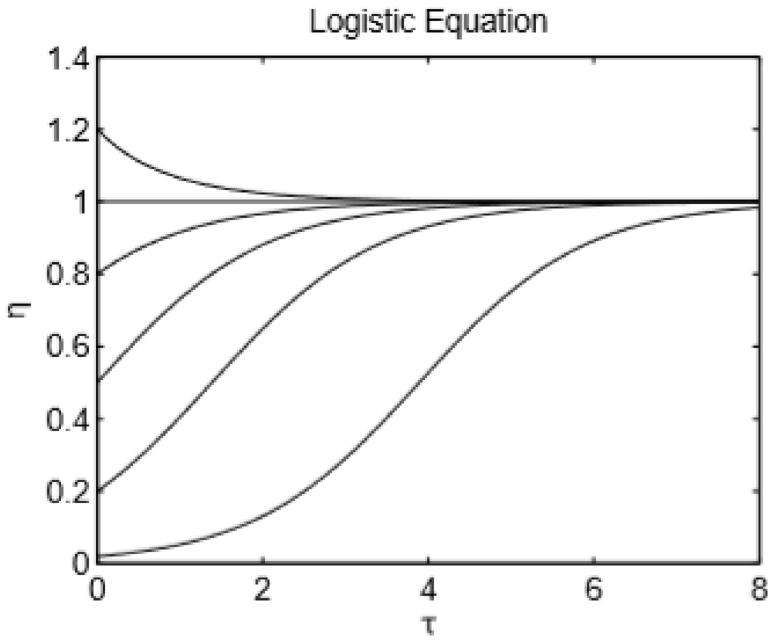

FIGURE 3.3: Stable and Unstable Fixed Points of Logistic Model

3.3.2 Estimation of catch per unit effort (CPUE)

In fisheries and conservation Biology, the catch per unit effort (CPUE) is an indirect measure of the availability of a target population when it varies could indicate the changes in the number of the species. CPUE has a number of advantages over other methods of measuring abundance as the data can be easy collected and analyzed even for non-specialist. This means that decisions about stock management can also be made by the people doing the harvesting. A decreasing CPUE indicates over-exploitation, while a constant CPUE indicates sustainable harvesting.

The best practice is to standardize the effort employed (e.g. number of traps or duration of searching), which controls for the reduction in catch size that often results from subsequent efforts. Although CPUE is a relative measure of abundance, it can also be used to estimate the absolute abundance of population. The main difficulty of using

CPUE as a measure of abundance is to define the unit of effort.

CPUE $=\frac{Totalcatch}{Effort}$ CPUE $=\frac{30,760}{3200}$ = 9.61

3.3.3 Estimation of cachability coefficient

cachability is defined as the average proportion of a stock that is taken by each unit of fishing effort. q $=\frac{CPUE}{Dp}$ Where CPUE is the catch per unit effort and Dp is the fish stock density (population size). The value of q lies between 0 and 1 (0 being no catch and 1 being the entire stock), and typically will be very small.

3.3.4 The Carrying capacity of the Dam

The determination of the carrying capacity of a reservoir or distinct reservoir branches is an auspicious new approach in reservoir management, and is a key factor, which will allow the limited development of aquaculture[16].

According to [15] the carrying capacity is an important concept for sustainable management, to help to manage the upper limit of environment to produce fish culture so that can prevent accepted changes by the natural ecosystem.

Phosphorus is the basic restrictive element in lakes that can be measured to test a lake's trophic status and make assumptions as to its productivity, from which equations can then be used to determine carrying capacity[14]. The phosphorus budget model that was fist established by Vollen weider, and then developed by Dillon- Rigler, is widely used in estimating phosphorus concentration to calculate the biomass and productivity of all the biological components of the lakes[13].

Because the Dillon - Rigler model have the best predictive abilities in shallow lakes, deep lakes, and reservoirs in both temperate and tropical regions[14].The amount of feed used for fish production is given by the food conversion ratio(FCR)

$FCR = \frac{useddryfood}{totalfreshbodyweightoffish}$

and amounts to $1.3 - 1.5$[16]. Estimation method of total Phosphorus released from fish food into the environment[15] $E_p = FCR\dot{p}_{feed} - p_{fish}$ Where E_p P release to the environment

p_{feed} p content of fish feed in %)

p_{fish} p content of fish body in %)

According to the estimation of carrying capacity using the Dillon and Rigler phosphorus budget model has the following procedures.

1. p is flushing rate which is the theoretical water exchange time of the reservoir and given by $p = \frac{Q}{V}$ Where Q is the average total water volume out flowing each year V Volume of the reservoir

2. R: Phosphorus retention coefficient is calculated as $R = \frac{1}{1+0.747(P^{0.507})}$

3. X:is the net deposition of total P lost permanently to the sediments as a result of solids deposition. Usually the value of X is around $0.45 - 0.55$ (in the calculation using 0.5)

4. R_{fish}:The amount of Phosphorus produced by fish that endured by sediment permanently $R_{fish} = X + [(1 - X)R]$

5. The capacity of the water body for intensive cage Fish culture is the difference $[\triangle P]$ between $[P]$ prior to exploitation $[P]_i$ and the final desired/acceptable $[P]$ once fish culture is established, $[P]_f$.

Allowable increase in total P concentration $[\triangle P]$ from cage culture is calculated as follows $[\triangle P] = [P]_f - [P]_i$

$[P]_i$ is the critical p concentration and is probably between $20 - 40\mu gL$ John A. Hargreaves used $40\mu gL$. $[P]_f$

Maximum P allowed in waters[16]. Hargreaves used $100\mu gL$

6. L_{fish} is total P produced by fish culture given by $L_{fish} = \frac{[\triangle P]Z_p}{(1-R_{fish})}$

Where Z is mean depth of the reservoir (m) and is given by

$Z = \frac{V}{A}$, is the area of the reservoir in m^2.

7. L_a is the total acceptable p loading tolerated by the environment in one year. $L_a = L_{fish}\dot{A}$ Therefore. the carrying capacity is calculated as follows $CC = \frac{L_a}{E_p}$

3.3.5 Modeling Assumptions

- The fish population is considered as a single homogeneous species(Nile Tilapia).
- Environment factors (temperature, dissolved oxygen, pH) remain constant during the study period.
- The carrying capacity (K) is time-invariant and estimated via the phosphorous budget model.

- Harvesting is the primary anthropogenic stressor; natural predation and disease are negligible.
- The intrinsic growth rate (r) and catchability coefficient (q) are constant.
- Fishing effort is evenly distributed and proportional harvesting model.
- Migration into and out of Tekeze Dam is negligible.
- The Dam is considered a closed system for the purpose of population modeling.

3.3.6 Important Data and Parameter Determination

According to primary and secondary sources of data we get the following values of the parameters.

3.3.6.1 Important Data and Parameter Determination

According to the Dillon and Rigler model the carrying capacity of the Dam is calculated as follows

List of Data and thier values

serial no	Types of Data	Amount	Unit
1	The average total water volume out flowing each year	3750000000	m^3
2	The food conversion ratio of Nile Tilapia	1.7	
3	The maturity rate of the Nile Tilapia	80	%
4	P content of Nile Tilapia feed	0.98	%
5	P content in one tonne of Nile Tilapia body weight	0.75	%
6	Volume of the reservoir	9293000000	m^3
7	The area of the reservoir	160400000	m^2
8	The critical (initial) P concentration	40	μgL
9	Maximum P concentration allowed in water	100	μgL
10	catchability coeffiient	0.0015	

1. The amount of phosphorus released to the reservoir by one tonne of fish

$E_p = 1.7 \times 0.0098 - 1 \times 0.0075 = 0.00916$

2. The flushing rate of the reservoir $p = \frac{3750000000m^3}{9293000000m^3} = 0.40353$

3. The phosphorus retention coefficient of the reservoir $R = \frac{1}{1+0.747\times(0.40353)^{0.507}} = 0.67957$

4. The amount of Phosphorus produced by fish that endured by sediment permanently $R_{fish} = 0.5 + (1 - 0.5)\,0.67957 = 0.839785$

5. The capacity of the water body for intensive cage Fish culture $[\triangle P] = 100\mu gL - 40\mu gL = 60\mu gL$

6. Mean depth of the reservoir $z = \frac{9293{,}000{,}000m^3}{160{,}400{,}000m^2} = 58m$

7. Total P produced by fish culture $L_{fish} = \frac{60\mu gL(58m)0.40353}{1-0.839785} = 8,777.0392\frac{\mu mg}{L} = 8,777.0392\frac{mg}{m^2}$

8. The total acceptable p loading tolerated by the reservoir in one year $L_a = 8,777.0392\frac{mg}{m^2}\times 160,000,000m^2$

$= 1,404,333,120,000mg = 1,404.33312$tonne

9. The carrying capacity of the reservoir

$CC = \frac{1{,}404.33312}{0.00916} = 153,312$ tonnes

Chapter 4

Stability Analysis of Optimal Production and Harvesting of Fish in Tekeze dam

4.1 Stability Analysis of the Logistic Growth Model

In this section numerical simulation of the harvesting models were done using MATLAB command ode45 to solve all the differential equations. Detail stability analysis of each logistic growth models are incorporated.

4.1.1 Logistic Growth Model With Out Harvesting

Taking $r = 0.8$ and $K = 153312 tonne$. The equilibrium points of the following model were calculated as below:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) \tag{4.1}$$

To find the equilibrium points of the model, we let $\frac{dN}{dt} = 0$, this implies that
$rN(1 - \frac{N}{K}) = 0$
Thus, solving the value of N yields, $N = 0$ and $N = 153312$
Therefore, $N = 0$ and $N = 153312$ are equilibrium points of the model. From this, the logistic model without harvesting, if the initial population of the fish started with $N = 0$,

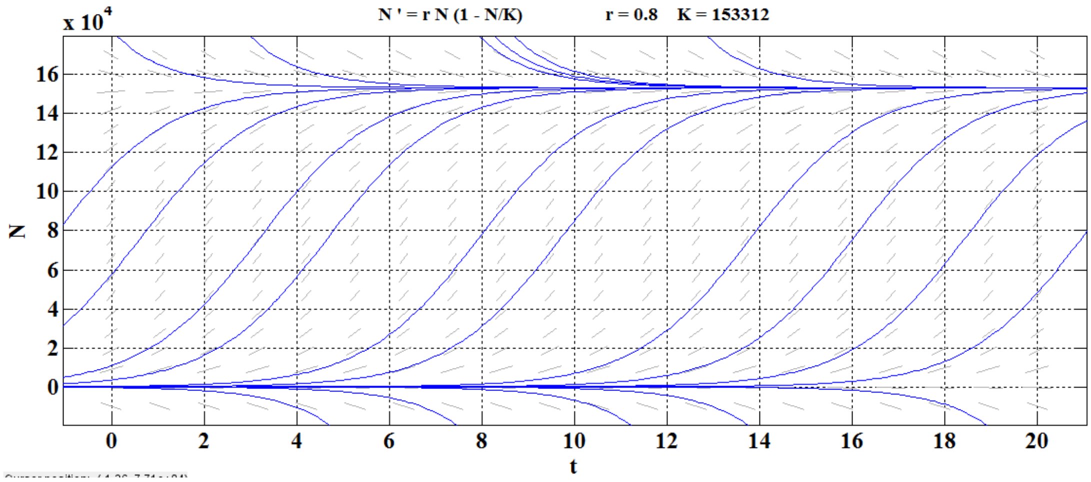

FIGURE 4.1: Direction field and solution curve of Logistic growth without harvesting

the population of the fish remains at $N = 0$.

Similarly, If the initial population of the fish is started with $N = 153312$ the population remains at the same level. The equilibrium point $N = 0$ is an unstable, because the solutions near this point were repelled or asymptotic. This means when initial population N_0, is just above and below $N = 0$, the fish population grows away from $N = 0$. However, the equilibrium point at $N_0 = 153312$ is stable, because solutions near this point are attracted to it. This means given an initial fish population in the interval (0,153312), the population increases to $N = 153312$ and remains at the same level. But, if No is greater than the carrying capacity 153,312 the fish population declines and approach a limiting value 153312 as shown in figure (4.1).

4.1.2 The Logistic Growth Model with constant harvesting rate

The Logistic Growth Model with constant harvesting is given by:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - H \tag{4.2}$$

where the values of $r = 0.8$, $K = 153312$ and H is constant to be calculated.

To determine the equilibrium points for constant H, we have:

$0.8N\left(1 - \frac{N}{153312}\right) - H = 0$

$0.8N - 0.00000521812N^2 - H = 0$

Equivalently, $-0.8N + 0.00000521812(N)^2 + H = 0$

$$N = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000521812)H)}}{2(0.00000521812)}$$

Rearranging and assuming the expression under the square root (discriminator to be zero, then $H = 30,662 tonne$ The value H = 30,662 tonne is called the maximum sustainable yield (MSY) which is the total allowable catch that can be harvested from the dam. The value $H = 30,662 tonne$ is also called the bifurcation point. Consider the following three values of harvesting:

$H = 30,662$

$H > 30,662$

$H < 30,662$

Case I: For $H = 30662 tonne$, we have one equilibrium point.

Thus,

$$N = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000521812)H)}}{2(0.00000521812)}$$

$N = 76,656$

The following figures illustrate this numerical analysis:

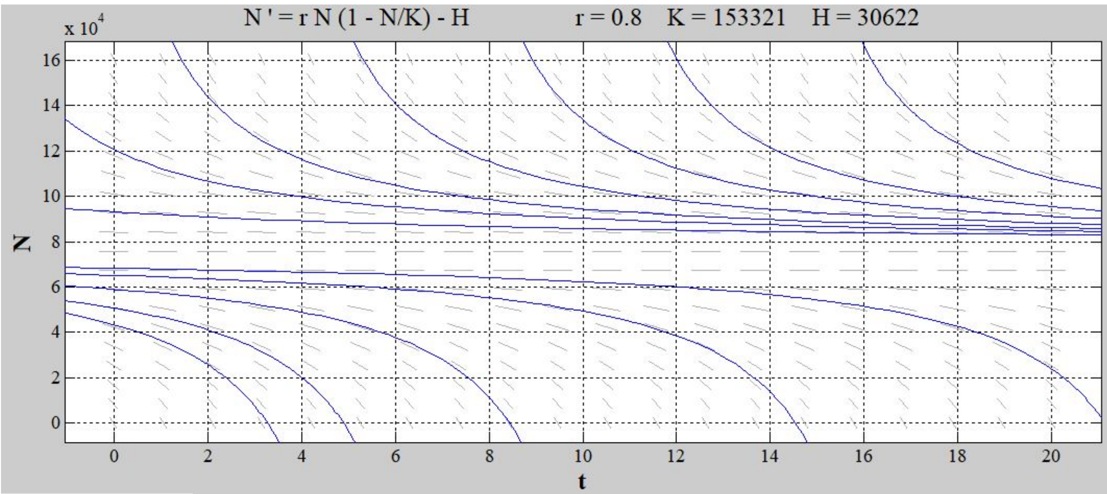

FIGURE 4.2: Direction field and solution curves of constant harvesting $H = 30,662$

From Figure(4.2), When $H = 30,662$, we can see that if the initial population of fish N_0 is greater than 76,656; the population of the fish will decrease and approach to 76,656. Similarly, for initial population of N_0 fish lower than 76,656; the population of the fish will decrease and finally gets to extinction.

Case II: when $H < 30,662$

$$N = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000521812)H)}}{2(0.00000521812)}$$

,

two equilibrium points exist (i.e 65389 and 87,923). From (4.3) The upper equilibrium point(N = 87,923) is stable because the solution is attracted to the equilibrium point. The lower equilibrium point($N = 65,389$) is unstable because the solution near this point is repelled. Thus, we can conclude that the harvest could not be too large without depleting the fish resource.

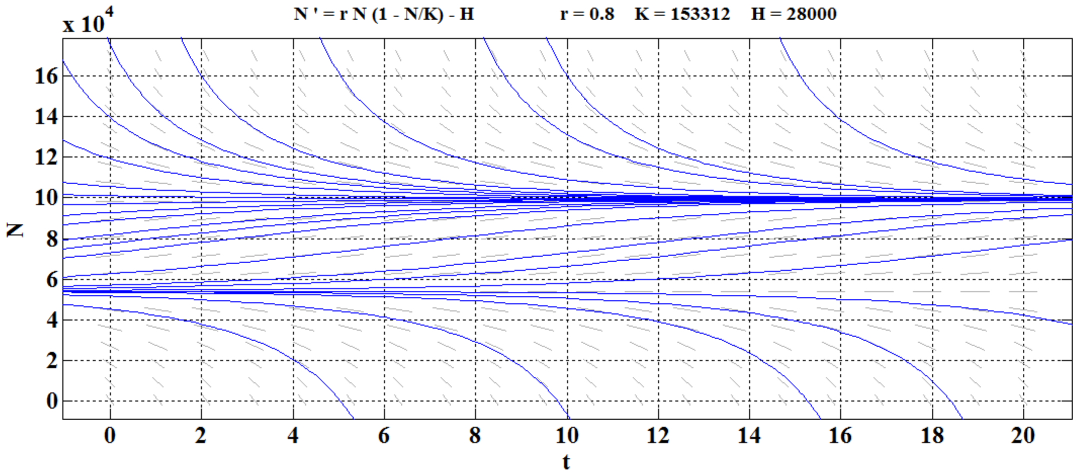

FIGURE 4.3: Direction field and solution curves of constant harvesting $H = 28,000$

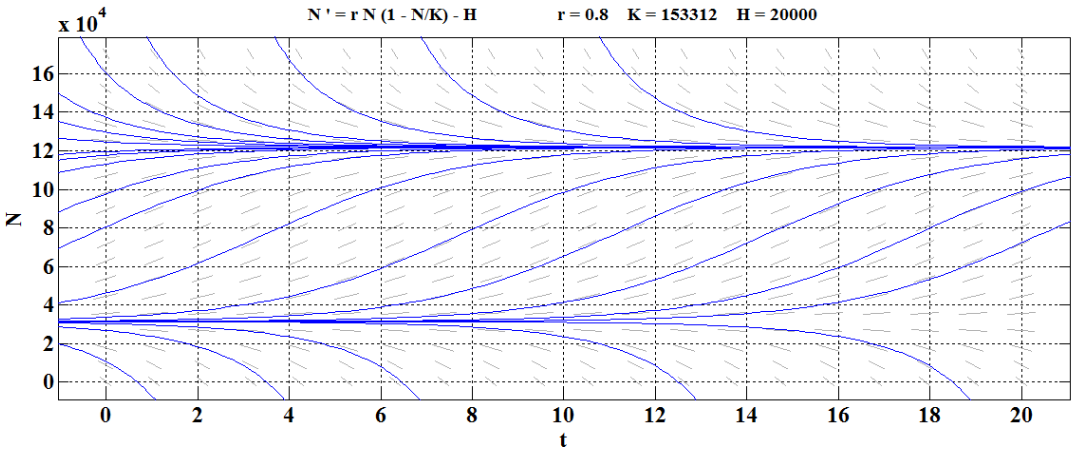

FIGURE 4.4: Direction field and solution curves of constant harvesting $H = 20,000$

$$N = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000521812)H)}}{2(0.00000521812)}$$

As the same fashion as the above (Figure 4.3) this has two equilibrium points(i.e 31453 and 121,859). From figure(4.3) The upper equilibrium point($N = 121,859$) is stable because the solution is attracted to the equilibrium point. The lower equilibrium point($N = 31453$) is unstable because the solution near this point is repelled. Thus, we can conclude that the harvest could not be too large without depleting the fish resource.

Case III: For $H > 30,662$.

$$N = \frac{-(-0.8) \pm \sqrt{(-0.8)^2 - 4(0.00000521812)H)}}{2(0.00000521812)}$$

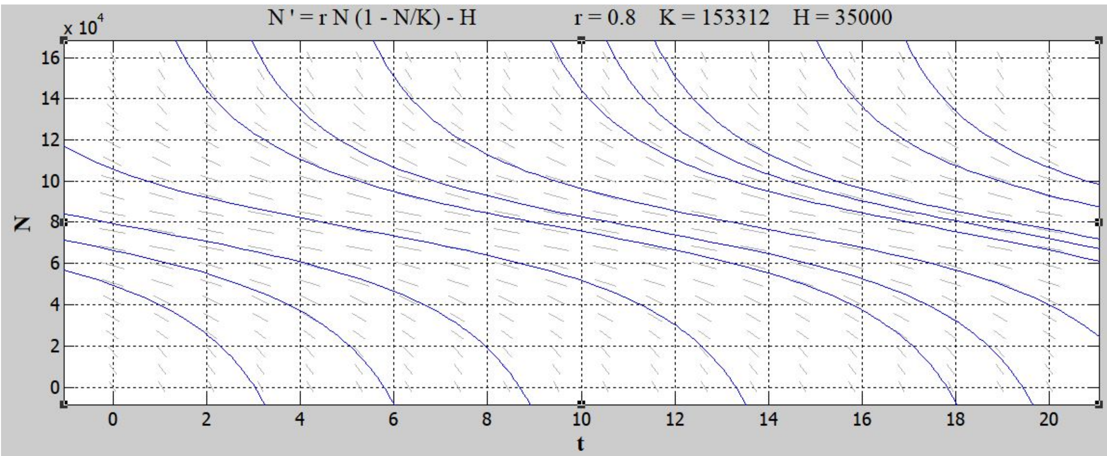

FIGURE 4.5: Direction field and solution curves of constant harvesting $H = 35,000$

Figure(4.5) shows a decreasing trends of fish population. This means that if we continue to remove this amount of fish from the dam, the population will go to extinction independent of the initial population size

4.1.3 The Logistic Growth Model with Proportional rate of harvesting

The Logistic Growth Model with proportional harvesting is given as:

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - mqEN \tag{4.3}$$

Equilibrium points of the differential equation occur when the growth rate of the population is equal to the harvest rate (i.e, $rN(1 - \frac{N}{K}) = mqEN$).
$N_1 = 0$ and $N_2 = K(1 - \frac{mqE}{r})$
Using the parameters values $r = 0.8$, $m = 0.01$, $q = 0.0015$ and $K = 153312$, the following figures illustrate the numerical analysis with fishing effort being varried.
As shown from figure(4.6-4.8) if the population starts above N_2, harvest rate is greater than growth rate, thus the fish stock decrease back to the equilibrium point.

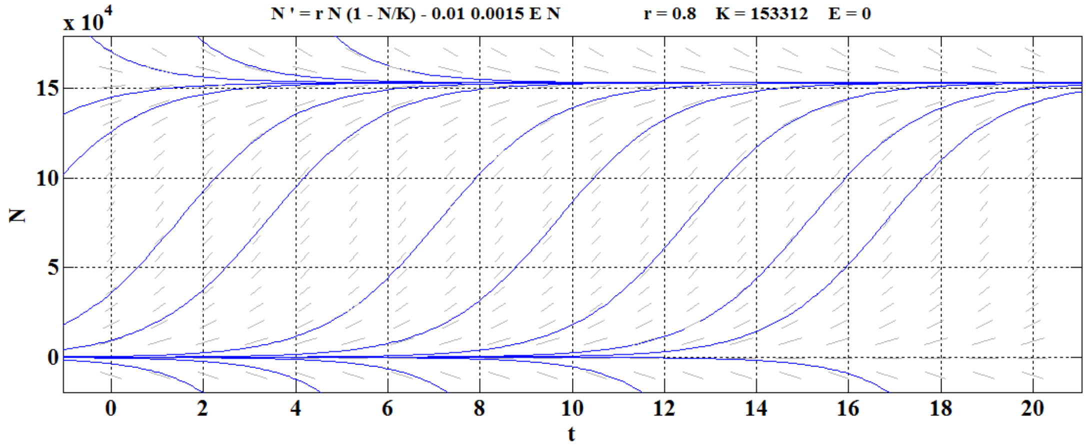

FIGURE 4.6: Proportional harvesting with $E = 0$

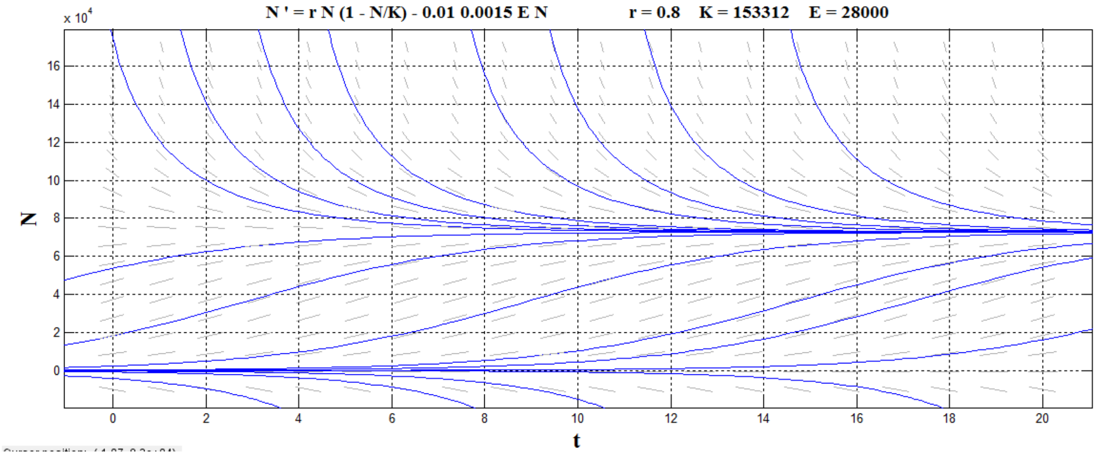

FIGURE 4.7: Proportional harvesting with $E = 28000$

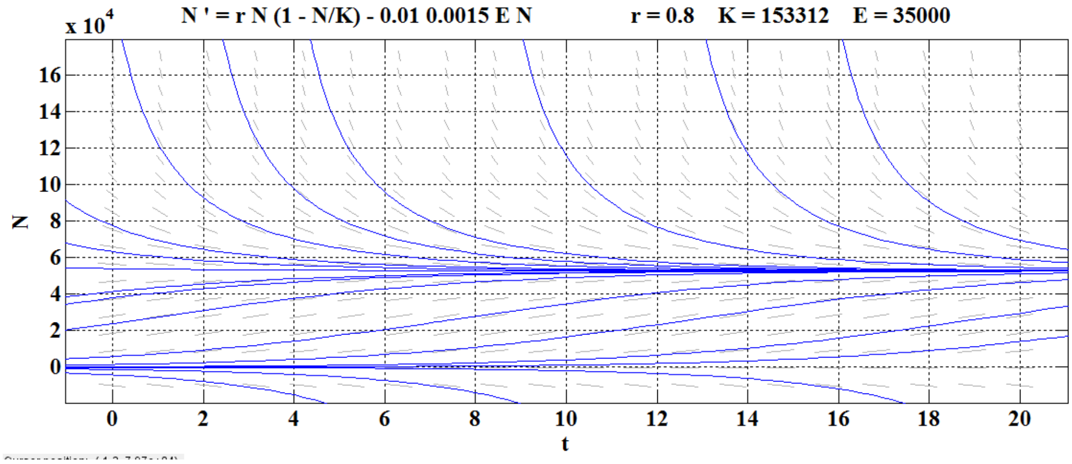

FIGURE 4.8: Proportional harvesting with $E = 35000$

If the population starts above N_2, harvest rate is greater than growth rate, thus the fish stock decreases back to the equilibrium point. If the population is below N_2, harvest rate is less than growth rate, then the population rises to the equilibrium N2. In this case, since the equilibrium N_2 is stable it is sustainable, then in turn extinction is unstable. Consequently, as fishing mortality (qE) increases the equilibrium N_2 decreases and shifts to the left (4.8). Eventually, for large enough values of fishing mortality, and N_2 has gone far enough to the left that harvest exceeds growth for all positive population levels. Then $N_1 = 0$ is the stable equilibrium. Moreover, since catchability coefficient is fixed (it is a constant), then increased fishing effort will lead the population go extinct.

4.1.4 The Logistic Growth Model with Periodic rate of harvesting

From The logistic growth model with periodic rate of harvesting, we have the following system of differential.

Taking, $r = 0.8$, $K = 153,312$ and allowing the harvesting H to be varied.

$$\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - h\,(1 + sin(2\pi t)) \tag{4.4}$$

We can also discuss the local stability of the equilibrium points by studying some numerical simulations as below

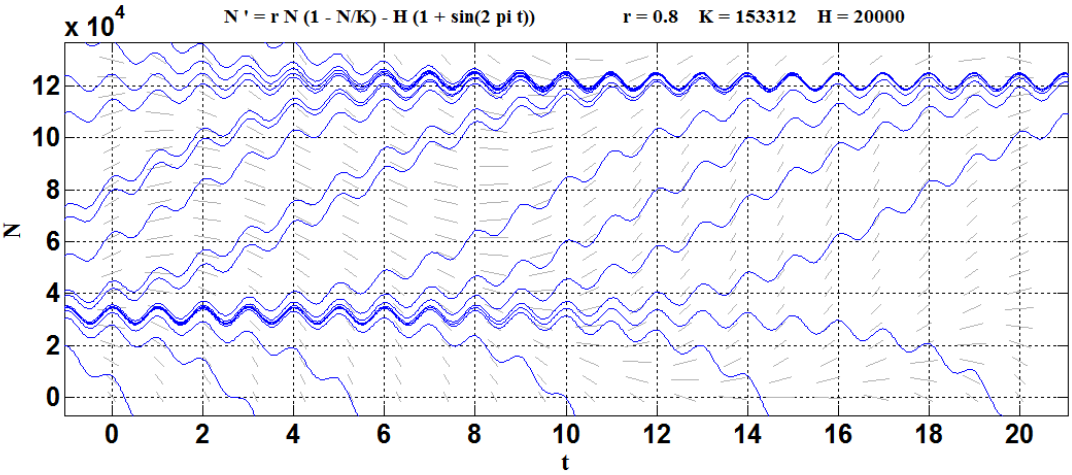

FIGURE 4.9: Solution curves of periodic harvesting with $H = 20000$

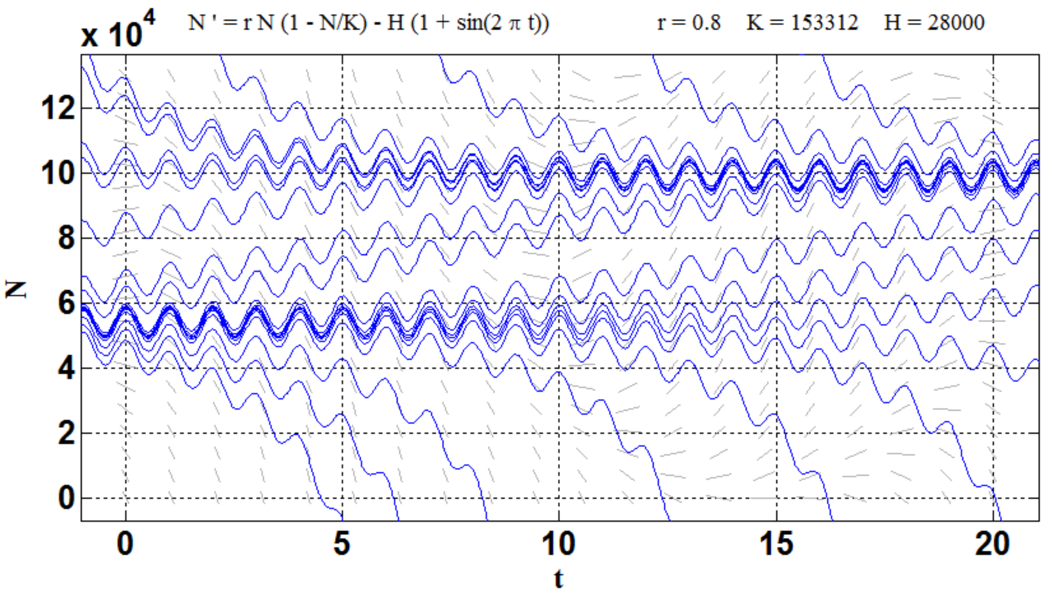

FIGURE 4.10: Solution curves of periodic harvesting with $H = 28000$

From figure (4.9), there are two periodic solutions that oscillate about the equilibrium values when $H = 20000$. The solutions converge to one periodic solution that oscillates around the stable equilibrium point. In figure (4.10), shows that if $H = 28,000$ are harvested, there is only one equilibrium point and this periodic differential equation has the same bifurcation value as the logistic differential equation with constant harvesting.

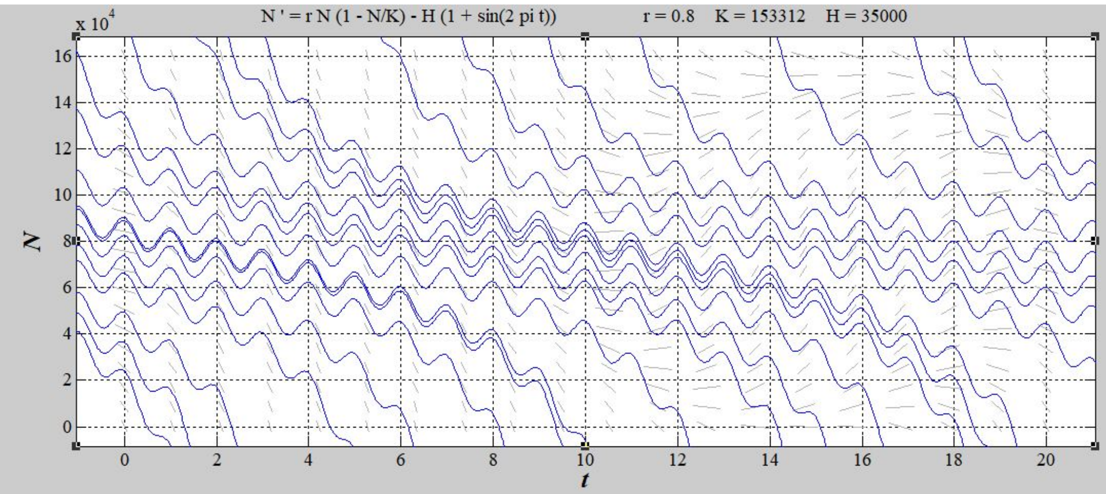

FIGURE 4.11: Solution curves of periodic harvesting with $H = 35000$

Moreover, figure (4.11), if we allow the fishermen to harvest even more (i.e $H = 60,000$), we go past the bifurcation value and the population will go extinction no matter what time we wait.

4.1.5 Maximizing Fish Yield

The main goal of this particular study was implementing various fish harvesting strategies to maximize catch in a sustainable manner. In fishery management, it is important to fish in such a way that a species is sustainable and not getting the risk of becoming extinct. To this end we need not to over harvest fish with small populations density. This fact suffice to show how population conditions greatly impact the dynamics of the system. Let's examine the effect of varying harvest rate to maximize the yield for populations that are not at equilibrium. The optimal harvesting rate in the optimal fish yield discussed above was numerically studied as follows Consider the one-species population growth model, where $N = N(t)$ governed by the differential equation $\frac{dN}{dt} = rN\left(1 - \frac{N}{K}\right) - hN$

Where, h = mqE and $N(0) = N_0$, the total fish yield as a function of h is given by $Y_T(h) = hN(t), dt$, for $T > 0$ (integral from 0 to T). The total fish yield Y as a function of T and h is defined by

$$Y_T(h) = \frac{Kh}{r} \ln[\frac{rN_0(e^{(r-h)T} - 1)}{K(r-h)} + 1]$$

We need to solve the optimal rate of harvesting h_T numerically for which the total yield is maximized.

A MATLAB code was used to numerically find the harvest rate that maximizes yield over time of $T = 5, 10, 15, 20$ units and plot yield as a function of harvest (figure 4.13-4.14). Fish yield changes its value indefinitely as time passes. In addition, from figures (figure 4.13- 4.14), the rate of harvesting h_T approaches the value $\frac{r}{2}$ as time passes. It also shows how much the initial population density affected the average maximum fish yield and to what extent maximum harvest rate depends on the initial conditions. Moreover, figure (4.14) shows that the fish yield increases as the initial population increases. For this reason, one can say that the fishermen should harvest at a rate of $h = \frac{r}{2}$ so that maximum yield is obtained.

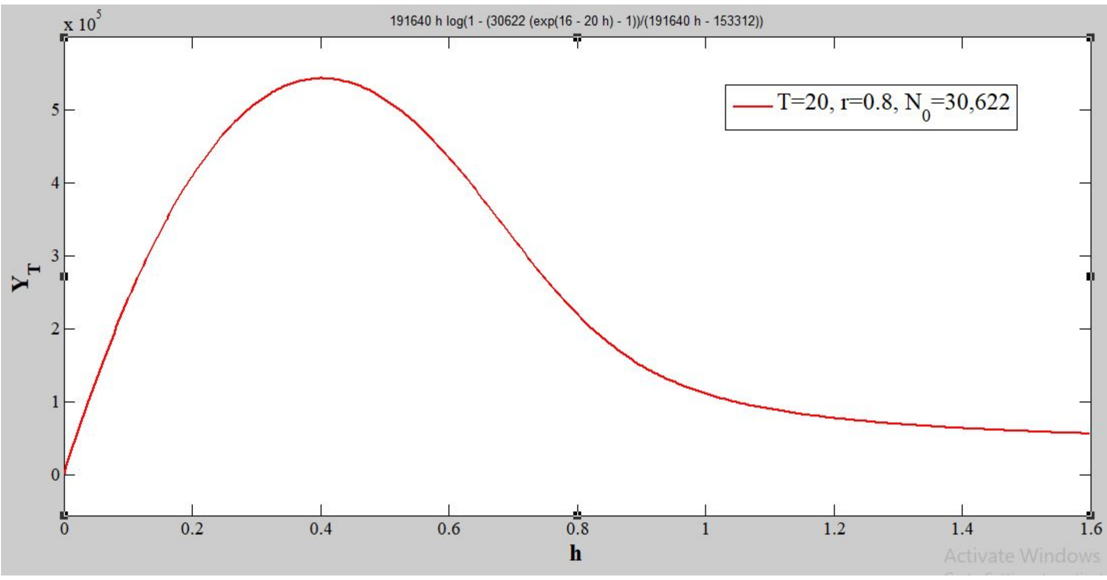

FIGURE 4.12: Fish Yield after $T = 20$

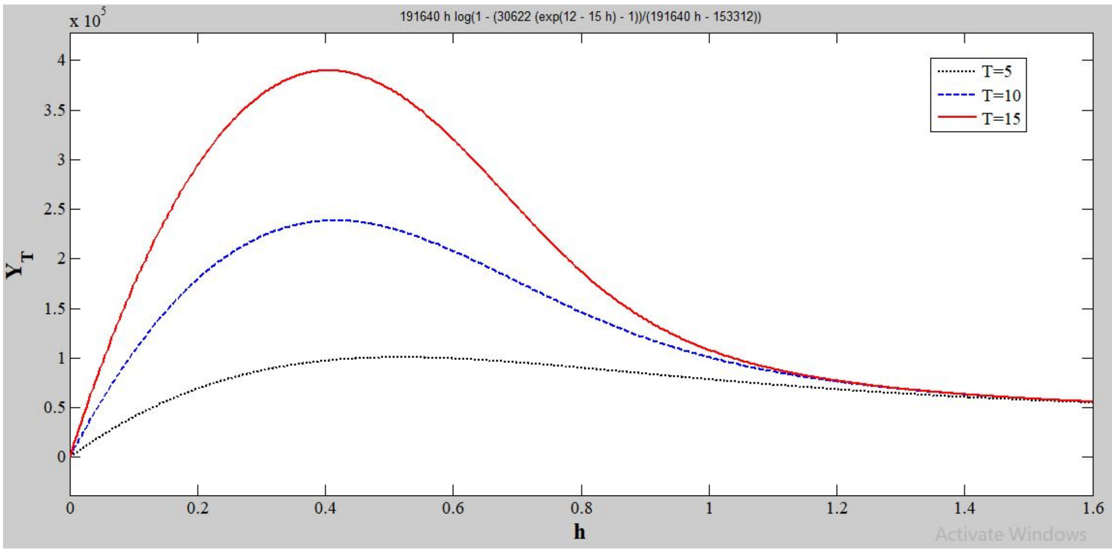

FIGURE 4.13: Fish Yield after $T = 5, 10, 15, when, N_0 = 30,662$

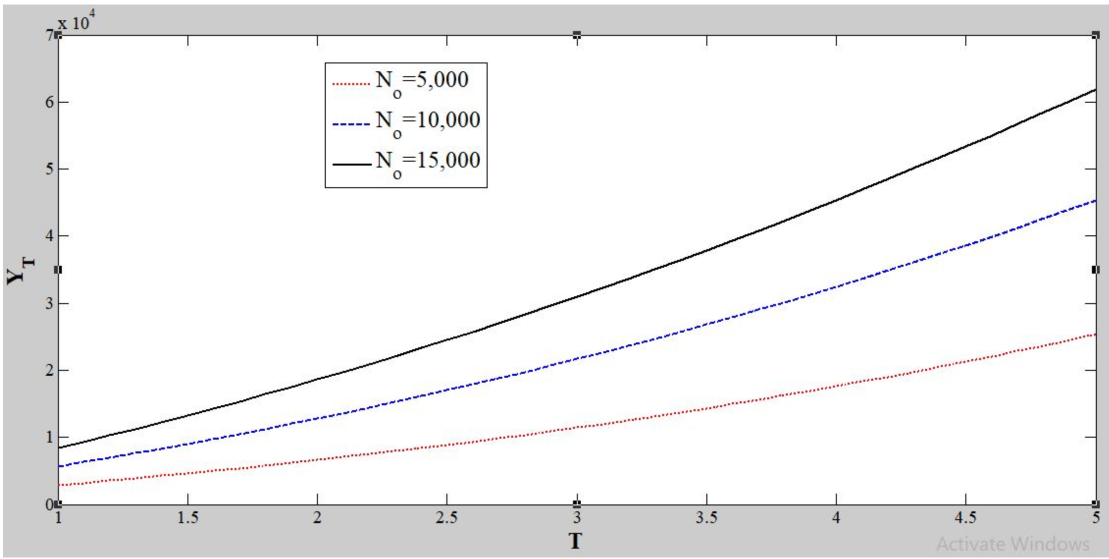

FIGURE 4.14: Fish Yield for $N_0 = 5,000, 10,000, 15,000, when, h = 0.4$

Chapters/Chapter5

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