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COMPARATIVE PERFORMANCE ANALYSIS OF LQR, PID, FUZZY-PSO AND PSO-PID CONTROLLERS ON QUARTER CAR ACTIVE SUSPENSION SYSTEM

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COMPARATIVE PERFORMANCE ANALYSIS OF LQR, PID, FUZZY-PSO AND PSO-PID CONTROLLERS ON QUARTER CAR ACTIVE SUSPENSION SYSTEM

A Thesis

Submitted In Partial Fulfilment of the Requirements for the Award of Degree

Of
MASTERS OF SCIENCE
In
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By

KOKEB GEBREMEDHIN

Under the Supervision
of

ATKLTI EYASU (PHD)

September, 2024

Mekelle, Ethiopia

DECLARATION

In partial fulfilment of the requirements for the award of a Master of Science degree, I hereby declare that the work submitted in the thesis entitled "**Comparative Performance Analysis of LQR, PID, FUZZY-PSO AND PSO-PID controllers on quarter car active suspension system**" Ethiopian Institute of Technology-Mekelle at the Mechatronics Engineering Chair, School of Mechanical and Industrial Engineering, is original to me and has not been presented elsewhere. All sources of materials used for the study have been duly acknowledged.

Student Name**:**	Signature	Date
Kokeb Gebremedhin Araya	--------------------	-----------------

I hereby certify that I have read this thesis prepared by kokeb Gebremedhin under my supervision recommend to be accepted as it fulfilled the request thesis standards to my knowledge.

Advisor name**:**	Signature	Date
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CERTIFICATION OF APPROVAL

This is to certify that that **Mr. Kokeb Gebremedhin Araya** has incorporated all comments forwarded by the internal, external examiners and chairperson during the thesis defense.

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ABSTRACT

The purpose of this study is to evaluate and compare the effectiveness of various control strategies for active suspension. MATLAB/SIMULINK software is used for both the controller design and the quarter vehicle model. The control strategies PID, LQR, PSO-PID, and FUZZY-PSO are employed. The suspension travel response and sprung mass acceleration response two critical parameters for ride comfort and road handling are chosen, examined, and compared between the responses of the active suspension system and the passive suspension system in order to assess the effectiveness of the vehicle's suspension system. The two Key Performance Indicators (KPI_{MAX} and KPI_{MAX}) are chosen to demonstrate a decrease in peak overshoot and a decrease in oscillation for the parameters chosen for the active and passive suspension system comparison. From this research, in summary, the PSO-PID and FUZZY-PSO controllers that were created exhibit exceptional performance in enhancing ride quality, hence increasing passenger safety and vehicle handling in the presence of two bumpy road disturbances. When it comes to decreasing to sprung mass acceleration and suspension travel, the PSO-tuned fuzzy logic controller performs the best, followed by PSO-PID, LQR, and PID controllers.

Key word*: Active suspension system, Quarter car model,* KPI_{MAX} *,* KPI_{MAX} *, PID, LQR, PSO-PID and FUZZY-PSO*

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LIST OF ACRONYMS AND SYMBOLS

List of acronyms

ASS	Active Suspension System
PSS	Passive Suspension System
SSE	State-Space Equation
LTI	Linear Time Invariant
RMS	Root Mean Square
MIN	Minimum value
MAX	Maximum value
PID	Proportional Integral Derivative
LQR	Linear Quadratic Regulator
PSO	Particle Swarm Optimization

list of symbols

m_{us}	Un-sprung mass (mass of the tire)
m_s	Sprung mass (mass of the vehicle body)
k_s	Spring constant for the sprung mass
b_s	Damping coefficient for the sprung mass
F_a	Actuator force
k_{us}	Tire stiffness coefficient
b_{us}	Tire damping coefficient
Z_{us}	Un-sprung mass displacement
Z_s	Sprung mass displacement
$\dot{Z}_{us}$	Un-sprung mass velocity
$\dot{Z}_s$	Sprung mass velocity
$\ddot{Z}_s$	sprung mass acceleration

$\ddot{Z}_{us}$	Un-sprung mass acceleration
Z_r	Road input displacement
Z_{r}_d	road input velocity
f_{ks}	Spring force for the sprung mass
f_{bs}	Damping force for the sprung mass
f_{kus}	Spring force for the un-sprung mass
f_{kt}	Spring force for the tire

CHAPTER ONE

1. INTRODUCTION

This chapter includes background of study which is basically on suspension system, statement of problem which typically answers why this research is done and what are the gaps to be fixed by this research paper, objective of the study which describes the general objective as well as specific objective of the study, the significance and the scope of the study and finally the organization of this study is included.

1.1 Background of the study

The vehicle's suspension system is a crucial and important part that, by absorbing vibrations from uneven roads under various driving circumstances, contributes significantly for both road handling and ride comfort or quality. A car's suspension system reduces the impact of irregular roads and road disruptions to give a smooth and comfortable ride. It is essential for preserving the handling, traction, and stability of the car in a variety of driving situations. Components of suspension system contains Shock absorbers, also known as dampers which regulate the springs oscillation, which helps to reduce vibrations and preserve tire contact with the pavement[1]. This system, which focuses on one-fourth of the suspension system of the car, usually consists of a control unit, actuators, and sensors. The control unit processes the real-time data that the sensors collect about the vehicle's vertical movement and imperfections in the road. In order to maximize ride quality and handling, the actuators actively adjust the suspension settings, such as shock absorber stiffness or damping, based on the input. Passive suspension system (PSS) is the most popular and conventional method of suspension system design it is an uncontrolled suspension system that relies on a spring and damper system. Active suspension systems (ASS) for quarter cars significantly reduce the impact of road disturbances and improve the vehicle's response to them, improving handling and comfort. This technology contributes to an upgraded driving experience marked by improved comfort and stability. It finds applications in luxury and high-performance vehicles as well as specialist automotive segments.

The ultimate in suspension technology is an active suspension system (ASS), which provides real-time changes that allow for unmatched control over ride quality and vehicle dynamics. In an active suspension system, sensors that track road conditions, vehicle movement, and driver input are combined with actuators, such as electromagnetic or hydraulic devices[2].

Vehicles model can be categorized into three groups: quarter car, half car, and full automobile models. In comparison to half and full car models, a quarter car model is the most basic type of car model; it only considers vertical motion when calculating the vehicle's performance. Vehicle roll and pitch motions, on the other hand, are not considered

Figure 1.1 shows quarter car active suspension system. which, at least in the early phases of vehicle dynamics design, are widely employed in automotive engineering because to their simplicity and ability to deliver qualitatively accurate information. Active quarter suspension significantly reduces suspension travel across a larger frequency range and dampens vibrations from the passenger seat, sprung mass, and un-sprung mass.[3].

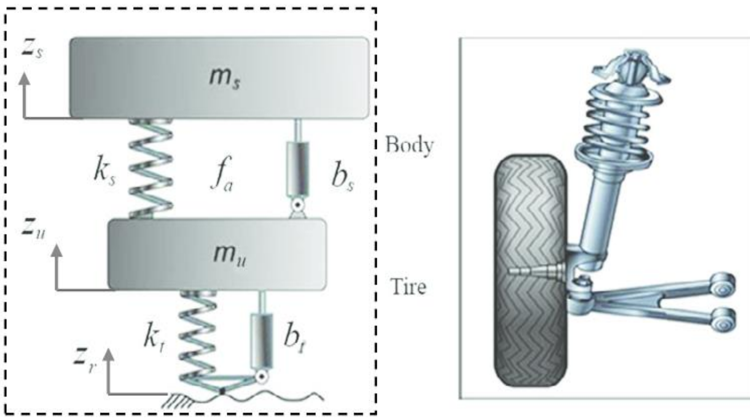

Figure 1.1 quarter car active suspension system

The half-car model which is shown in figure 1.2 makes it possible to analyze the suspension systems separately on each half of the vehicle by splitting the car into the front and back halves.

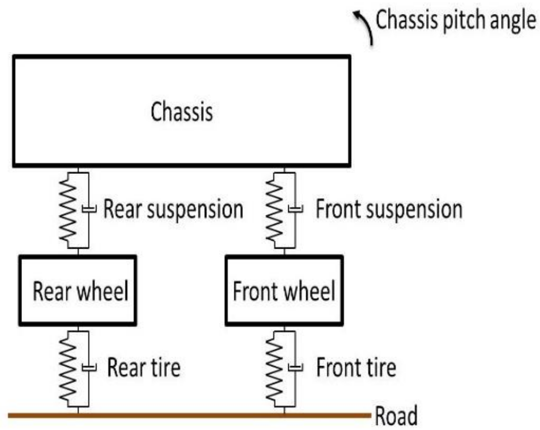

Figure 1.2 half car suspension system

Figure 1.3 shows full car model which integrates the front and rear suspensions into a single model of the entire car. considers every component of the suspension system, such as the control arms, anti-roll bars, dampers and springs. provide a comprehensive understanding of the dynamics of the car, including how the front and rear suspensions interact. provides a thorough examination of handling, stability, and ride comfort while taking the whole behavior of the vehicle into consideration.

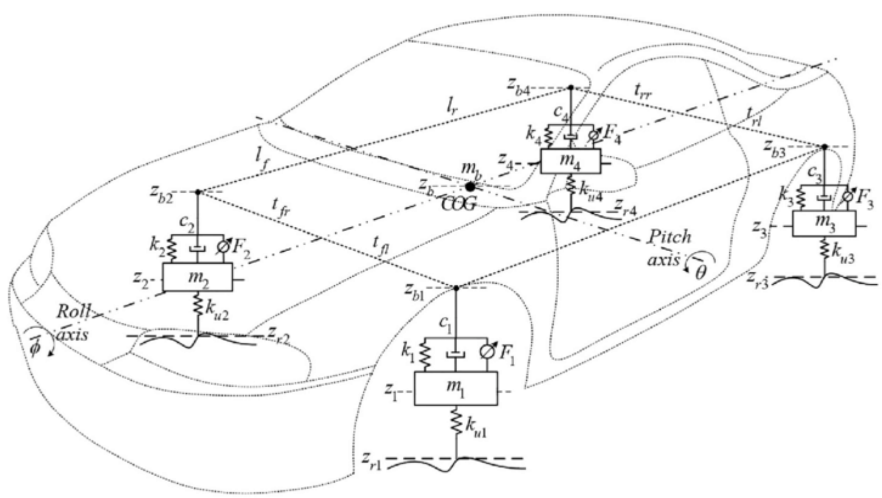

Figure 1.3 full car suspension system

It is challenging and time-consuming to analyze the behavioral response of the entire vehicle to different road imperfections while it is moving. To address desired Ride comfort and vehicle handling difficulties, a variety of numerical control techniques, including PID, fuzzy-PID, LQR, fuzzy-LQR, and sliding mode controller, have been developed. The outcomes of vibration suppression in suspension, sprung mass, and un-sprung mass systems differ according to different control mechanisms that have been devised and put into practice. Through simulation results, they are related to vehicle handling concerns and passenger ride comfort when compared to a passive vehicle.

1.2 Problem Statement

The suspension system is a crucial component of the vehicle's setup that improves road holding ability and ride comfort or quality, which in turn improves passenger safety and health. A quarter car model is used for a comparative performance evaluation, which considers the direction of vibrations acting vertically on the vehicle body. To manage active suspension, PID, LQR, PSO-PID, and FUZZY-PSO controllers are chosen. This model has already been the subject of several studies employing fuzzy logic control, sliding mode control, fuzzy-PID control method, PID, LQR, and LQG. This paper proposes the FUZZY-PSO suspension system control method, which is rarely applied to active suspension systems. The two inputs sprung mass displacement (error), sprung mass velocity (change in error), and output (actuator force) are tuned using P, S, and O parameters. This means that the research will determine the optimal values of the inputs and outputs by using the Particle Swarm Optimization technique to find the objective function. Additionally, a comparative study is conducted to determine the performance difference between the suggested controllers. Since we are seeking good ride comfort, road holding capability, designing of the above controllers under random road input profile and analyzing the performance of those controllers makes the research interesting in determining control techniques to provide better performance with respect to tire deflection, suspension travel, sprung mass displacement, un-sprung mass displacement, sprung mass velocity and acceleration, peak overshoot and oscillation reduction response. This research also focuses on finding the gap between the proposed controllers by comparing performance analysis of the controllers on quarter car active suspension system response by using MATLAB/SIMULINK software.

1.3 Objectives of The Research

1.3.1 General Objective

The objective of the research is evaluating the comparative performance analysis of PID, LQR, FUZZY-PSO and PSO-PID Controllers on quarter car active suspension system using MATLAB/ SIMULINK.

1.3.2 Specific Objective

- ✓ Design and tune the PID controller using conventional methods and evaluate its performance.
- ✓ Design and implement an LQR controller based on a linearized model of the active suspension system, optimizing the weighting matrices to achieve desired performance.
- ✓ Develop a PSO-PID controller, where the PID gains are optimized using the Particle Swarm Optimization (PSO) algorithm, and compare its performance to the conventional PID and LQR controllers.
- ✓ Design a Fuzzy-PSO controller, combining fuzzy logic for adaptive control with PSO for parameter optimization, and evaluate its performance against the other controllers.
- ✓ Assess and compare the controllers' performance in maintaining tire dynamic load within acceptable limits to ensure road holding.
- ✓ Evaluate and compare the controllers' ability to minimize suspension deflection (working space of the actuator) under the same road disturbance conditions.
- ✓ Quantitatively compare the performance of PID, LQR, PSO-PID, and Fuzzy-PSO controllers in reducing vehicle body vertical acceleration

1.4 Significance of the study

- This research helps determine which control strategy most effectively reduces vibrations and accelerations.
- This research evaluates how each controller manages tire dynamic load, contributing to safer driving conditions.
- The research provides insights into which controller offers the best balance between ride comfort and suspension travel.
- It investigates the effectiveness of optimization algorithms like PSO in tuning controller parameters, leading to improved performance.

- The research surrounding Fuzzy-PSO controllers allows for the furthering of knowledge surrounding the implementation of artificial intelligence within vehicle control systems.
- It contributes in increasing both the vehicle's body and passenger safety in a driving on uneven road surface.
- It advances automotive engineering technology by providing insight into the suspension system's vehicle dynamics control algorithm.
- It prolongs the life of automobiles and provides a response to consumer inquiries regarding how readily occupants might experience vibration from the suspension.
- It aids in launching additional research on suspension system other parameters by researchers.

1.5 Scope of the study

This research is aimed to study comparative performance analysis of different control strategies for an active quarter car suspension system by collecting different data from literatures, gathering information directly and indirectly from researchers and using MATLAB/SIMULINK software a simulation result is obtained and discussed briefly to select and implement better performed control strategies and to give insight for researchers for future works

1.6 Organization of the study

This research paper is organized into five chapters, as labelled clearly in the following way

- **Chapter One:** Introduction; Includes background of the study, the overall clue on the research, statement of the problems, the significance of the study and the scope of the study
- **Chapter Two:** Literature review; Deals with a summary of related works which are done previously on the active suspension system
- **Chapter Three:** Material, methodology and analysis; In this chapter the methodology used to study the research, the modelling and the governing equation of motion for quarter car active suspension system and the design and performance analysis of proposed controllers are generated
- **Chapter Four:** result and discussion; In this section the simulation result from MATLAB/Simulink software for the selected controller on quarter car active suspension system and the performance analysis of those controlled on improving passenger safety

and road holding ability is compared to each other and with passive suspension system is presented and discussed briefly

- **Chapter five:** conclusion, recommendations and future work are described.

CHAPTER TWO

2. LITERATURE REVIEW

2.1 Introduction

This chapter reveals review on the classification of suspension systems, the types of vehicle models that are designed and articulated in different years and related research publications and reviews on quarter car suspension system modelling and design of control strategies are described below.

2.2 Related research studies

Modified PID Controller to regulate vibration on quarter car active suspension system has been proposed [4]. In this study a modified PID controller is used to regulate and decrease vibration and to lessen the erratic movement active quarter car suspension systems model for improving vehicle stability and ride comfort. according to the study, the performance of the active suspension system improved by regulating and controlling the hydraulic actuator's action and system fluctuation and vibration were decreased by using the proposed controller, which also had superior performance than the conventional PID controller. Additionally, the research reveals computing gains of modified PID controller is easier with respect to the time taken to compute fuzzy and Fuzzy-PID controllers. This work develops a vertical planar oscillation model for a quarter car vehicle model using the LQR active suspension control technique [5]. In the research, the LQR control approach is employed to evaluate the performance of an active suspension system and compared with passive suspension system. According to the comparison result an active suspension system shows a reduction in amplitude, oscillation damping, and vibration on the vehicle body that of the passive suspension system. Study on an active quarter model simulation using MATLAB is conducted [6]. The research paper demonstrates how a stable, reliable, and controllable PID system can improve the Passenger Ride Comfort, Vehicle Stability, Safety, and road Holding capacity of an active quarter car suspension system model in time domain as well as frequency domain. The paper's simulation results demonstrate that the proposed PID control technique performs significantly better than a passive suspension system at achieving the specified levels of ride comfort and passenger safety. study of using a 3 degree of freedom quarter car model that is controlled by a fuzzy logic controller to discuss the direction of vibrations acting on the human body in the translational vertical direction of the coordinate system. According to the study bump analysis, the proposed controller performs better than the passive suspension system in terms of achieving good ride comfort when it comes to the acceleration of the chassis, the acceleration of the seat, the displacement of the seat, the displacement of the chassis, and the road holding capability [7]. modeling, simulation and control of the half-car suspension system are carried out using MATLAB-SIMULINK on a four-degree-of-freedom half-car model with an active suspension system that is hydraulically actuated and includes pitch and heave motions of the vehicle's un-sprung mass as well as PID, fuzzy, and fuzzy-PID controllers. With the help of the suggested controllers, research has done to compare active suspension systems to passive suspension systems and to develop a hydraulically actuated active suspension system with the least amount of deflection and acceleration in the presence of road disturbances. The comparison results revealed that the Fuzzy-PID controller performed the best among the other two controllers [8]. Four degrees of freedom to model, simulate, and control the suspension system of a half-car used Lagrange method to apply a novel active suspension system design to a half-car model with six degrees of freedom. Half of a car model has the specified active suspension system installed, and both the vehicle system and the suggested controller are fully explained. It is explored how the proposed controller compares to a traditional PID controller in terms of body suspension, passenger acceleration, and energy usage. In order to maintain both the vertical motion of the vehicle's suspension and the acceleration on the driver and passenger, the developed active suspension system involves sensing the displacement of the front wheel. As a result, the study shows the oscillations settle quicker when there is an active suspension and it also proves that there is reduction of amplitude of the oscillations when the purposed controller is used [9]. The modeling of a quarter-car double wishbone suspension described in the study was done using the Sim-Mechanics toolbox to produce a physical model of the suspension and SIMULINK/ MATLAB program, to produce a mathematical model. A PID controller is used to manage the vertical body acceleration. The proposed model performs better when compared to the well-known quarter-car model in terms of managing vertical acceleration with a PID controller [10]. Linear Quadratic Regulator (LQR) and fuzzy PID control strategies [11] used in this research which aimed to improve the performance of a vehicle active suspension system and control the vibrations that occurred in a vehicle. Three parts input signals (actuator force and road profile), Controller part, and the suspension system model are used on modeling and simulating the quarter car model with the proposed controllers in order to get desired suspension performance. [12]This research aimed to design quarter vehicle active suspension control model subjected to vibration excitation from a random road profiles using fuzzy logic control and PID controller for quarter car vehicle model and comparison between passive and active suspension system with the proposed controllers is carried out. Using the MATLAB / SIMULINK toolbox, the performance of the fuzzy and PID controlled suspension systems is quantitatively assessed. It is discovered that the suggested controllers provide more ride comfort when compared to passive suspension. The active suspension system for the quarter-car model is controlled by a fuzzy self-tuning PID controller in the study [13]. The goal is to reduce the suspension working space of the sprung mass and its change rate to maximize driver comfort. [14] Using a linear quarter-car model and a linear quadratic regulator (LQR) controller, this study compares the passive and active suspension systems. The simulation findings demonstrate that the LQR controller reduces the sprung mass's working space in the suspension, resulting in an improvement in ride comfort.[15] this research designed three controllers which are LQR based fuzzy controller, Fuzzy PID controller and Linear Quadratic Controller (LQR) on 3 degree of quarter car model and comparison between the proposed controllers on active suspension system and passive suspension system is carried out in MATLAB/SIMULINK software result shows that Fuzzy-LQR controller based active suspension system gives well result and steadiness as compared to other active suspension system controllers and passive suspension model. [16] For decreasing the impact of uneven road surfaces on vehicles, linear matrix inequality and linear quadratic regulator controllers are developed in the research work utilizing a nonlinear quarter car active suspension system model with hydraulic actuator. The effect of a bumpy road surface is reduced using an improved linear quadratic regulator and gaussian distributed controllers; the paper does not consider the input signal from the road. Of the proposed controllers, the LQG performs better than the passive suspension system in terms of improving passenger ride comfort and road holding capability. Most of the controller proposed in the literatures are on improving road holding capability and ride comfort and almost all papers reviewed have positive results towards improving vehicle stability as well as ride comfort. In this research study comparative performance analysis between four different controllers on ASS. The controllers used are PID, LQR and PSO_PID and FUZZY-PSO controllers in determining control strategy to deliver better performance with respect peak overshoot reduction and oscillation reduction.

CHAPTER THREE

3. MATERIALS METHODOLOGY DESIGN AND ANALYSIS

This section presents the materials or software used to design and model quarter-car active suspension systems, the detail design of the selected controllers (PID, LQR, PSO-PID, and FUZZY-PSO), and the comparative performance analysis of the proposed controllers.

3.1 Materials

For vehicle model, controller design and simulation analysis software used is MATLAB /SIMULINK software package. MATLAB (an abbreviation for Matrix Laboratory) is matrix-based system software for developing program codes, modelling system using blocks by SIMULINK and to evaluate mathematical, scientific, and engineering calculations.

3.2 Methodology

To come up with the objective of this research stated in chapter one, first of all an extensive literature survey and secondary data collection methods have been done by taking different journals and books that are basics for getting knowledge on how to select the model of the system and how vibration reducing analysis using different controllers can be done to get the desired ride quality. After getting data and information from literatures and survey, secondly problem identification and pre-processing stage which includes selection of model to be controlled, selection of the controllers to be designed and physical representation vehicle suspension system. So that quarter car ASS is selected for this research. Thirdly solver execution is done by standing from the model and using physics laws the mathematical equation that describes the behavior of a quarter car active suspension system and the relationship between road inputs and the vehicle vertical dynamic outputs are derived mathematically. Finally post processing stage by using MATLAB/SIMULINK software methodology the model, design, comparative performance analysis of the proposed controllers and passive suspension system from simulation result are evaluated and discussed.

3.3 Quarter car active suspension system model

A quarter car active suspension system is a simplified form of automotive suspension system that can represent the dynamics of the entire vehicle including the tire deflection, the suspension travel and the sprung mass and un-sprung mass vertical motions at any one of the wheels from the four-wheel vehicles[17].

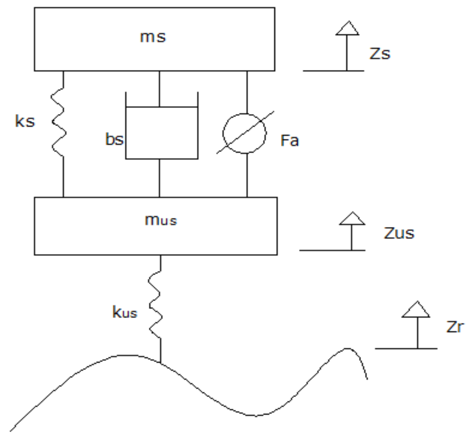

Figure 3.1 Two D.O.F quarter car active suspension system model

The quarter car active suspension system model, which has two mass, two springs, one damper, and one actuator, is shown in Figure 3.1. The first mass is the vehicle wheel mass, also known as the un-sprung mass(m_{us}) and the second mass is the vehicle body mass, also known as the body mass(m_s). A damper with a dampening coefficient of (b_s), an actuator force (F_a), and a spring with a constant stiffness coefficient of (k_s), are used to support the wheel and the vehicle body. Additionally, the tire has a stiffness factor (k_{us}) and no damping characteristics, therefore the damping factor (b_{us}) is equal to zero. The displacement (Z_{us}) and (Z_s). The state space variables of the system are displacement and the vertical displacement of the road which is the input (Z_r) represents the disturbance from the road.

3.4 Quarter car suspension system model specification and measurements

This research paper is done for light duty passenger vehicle and the parameters are selected from. A two D.O.F passenger vehicle model which only considers the vertical motion of the sprung mass as well as the un-sprung mass is used. Table 1 describes the values and the measurements for an active suspension system elements and state space variables used for analysis in MATLAB/SIMULINK software [18].

Table 1 parameter specification and measurements for light duty passenger vehicle (Agharkakli et.al 2012)

Elements	Symbol	Value	Units
Sprung mass	m_s	290	Kg
Un-sprung mass	m_{us}	59	Kg
Spring constant of suspension system	k_s	16812	n/m
Spring constant of wheel and tire	k_{us}	190000	n/m
Damping constant of suspension system	k_{us}	1000	ns/m
Damping constant of the wheel	b_{us}	0	ns/m

The following shortening assumptions were made during the mathematical modelling of the quarter-car active suspension system:

- The tire is considered only as a stiffness material or spring with no damping characteristics.
- Weight due to gravity neglected for the sprung and un-sprung mass
- The car is travelling at a steady speed in a horizontal direction on a straight road with a rough road profile.
- Wheel rotational motion is not considering
- only vertical motion of the body is considered the body.
- ideal joints are considered for connecting suspension system components.
- all springs are considered as linear
- The damper is considered as linear
- Both the sprung and un-sprung masses (the vehicle body and the wheels) are assumed to be uniform in mass.
- Bumpy road surface and unevenness are the main source of vehicle body vibration (road surface elastic deformation and engine vibrations are neglected).

3.5 Modelling and free body diagram for Two D.O.F quarter car model

Considering the model of quarter car active suspension given below, the free body diagram of two degree of freedom quarter car suspension system is done by taking the un-sprung mass and the sprung mass separately and putting the forces acting on both masses. When the vehicle is moving on a bumpy road three forces are acting on the sprung mass and four forces are acting on the un-sprung mass.

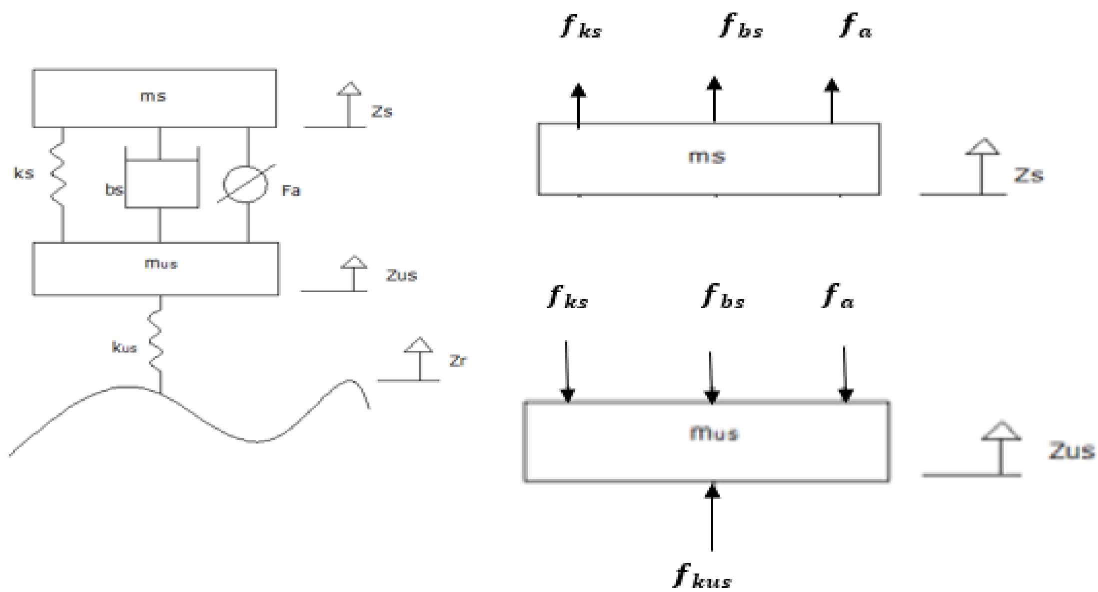

Figure 3.2 free body diagram for a two D.O.F quarter car active suspension system

3.6 Equation of Motion for ASS

- From free body diagram described in figure 3.2 we can define the equation of motion that related to the un-sprung mass and sprung mass are calculated as follows by using newton second law of motion ($f_{net} = ma$) to find the net force acting on both masses.
- For the **sprung mass**

$$f_{ks} + f_{bs} + f_a = f_{net(sprung\ mass)} \tag{3.1}$$

Where

$$f_{ks} = k_s(Z_{us} - Z_s) \tag{3.2}$$

$$f_{bs} = b_s(\dot{Z}_{us} - \dot{Z}_s) \tag{3.3}$$

$$f_{net} = m_s\ddot{Z}_s \tag{3.4}$$

Substituting equation (3.2), (3.3), (3.4) into equation (3.1) we can get:

$$m_s\ddot{Z}_s = b_s\dot{Z}_{us} - b_s\dot{Z}_s - k_s(Z_s - Z_{us}) + f_a \tag{3.5}$$

➢ For the **un-sprung mass**

$$f_{ks} + f_{bs} + f_{kus} + f_a = f_{net(unsprung\ mass)} \quad (3.6)$$

Where

$$f_{ks} = k_s(Z_s - Z_{su}) \quad (3.7)$$

$$f_{bs} = b_s(\dot{Z}_s - \dot{Z}_{us}) \quad (3.8)$$

$$f_{kus} = k_{us}(Z_r - Z_{us}) \quad (3.9)$$

$$f_{net(unsprung\ mass)} = m_{us}\ddot{Z}_{us} \quad (3.10)$$

Substituting equation (3.6), (3.7), (3.8), (3.9) into equation (3.5) we can get:

$$m_{us}\ddot{Z}_{us} = -b_s\dot{Z}_{us} + b_s\dot{Z}_s - k_s(Z_{us} - Z_s) - k_{us}(Z_{us} - Z_r) - f_a \quad (3.11)$$

3.7 State-Space Equation for ASS

State space representation refers to a mathematical model of a physical system that is described as a function of input, output, and state variables and is related by differential equations or difference equations of first order[19]. Within that space, the systems state can be depicted as a vector. In this research in order to analyze the quarter car active suspension system model the state space equation describing the model is created using the two equation of motion found in (3.5) and (3.11). For linear time-invariant continuous system the state space equation is given by:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (3.12)$$

$$y(t) = Cx(t) + Du(t) \quad (3.13)$$

By using the above form of Linear time-invariant continuous state space equation the state variable representing the system and the matrix is given as follows:

$$x_1 = Z_s - Z_{us} \quad (3.13)$$

$$x_2 = \dot{Z}_s \quad (3.14)$$

$$x_3 = Z_{us} - Z_r \quad (3.15)$$

$$x_4 = \dot{Z}_{us} \quad (3.16)$$

the fundamental dynamics characteristics of the active suspension system are captured by the state variable in the state space equation which offer thorough explanation of systems behavior throughout time. Then the vehicle is moving on bumpy road the displacement from the road profile (Z_r) is considered as an input to the system causes the wheel to deflect with a displacement of ($Z_{us} - Z_r$). Since the components are interconnected to each other the wheel deflection causes to a suspension to travel vertically with a relative displacement of ($Z_s - Z_{us}$). The time derivative of the respected displacement will result velocity therefore the suspension travel, the wheel deflection, the sprung mass velocity ($\dot{Z}_s$) and the unsprung mass velocity ($\dot{Z}_{us}$) are considered as the output of the system.

Now it is easy to write the state space model in matrix form by substituting the state variables

$$\dot{x}(t) = \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} \tag{3.17}$$

Where

$\dot{x}_1$: is velocity of the suspension travel

$\dot{x}_2$: is sprung mass acceleration

$\dot{x}_3$: is the un-sprung mass velocity

$\dot{x}_4$: is un-sprung mass acceleration

From the governing equations (3.5) and (3.11) the state matrix A is given as:

$$A = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{b_s}{m_{us}} & \frac{-(k_{us})}{m_{us}} & \frac{-(b_s+b_{us})}{m_{us}} \end{bmatrix} \tag{3.18}$$

By substituting the values of vehicle parameters form table 1 into equation (3.18) we get

A =

	x1	x2	x3	x4
x1	0	1	0	-1
x2	-57.97	-3.448	0	3.448
x3	0	0	0	1
x4	284.9	16.95	-3220	-16.95

From the governing equations the input matrix B is givens as:

$$B = \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_{us}} \\ -1 & 0 \\ \frac{b_{us}}{m_{us}} & \frac{k_s}{m_{us}} \end{bmatrix} \tag{3.19}$$

Substituting the values of vehicle parameters into equation (3.19) we get matrix B as;

B =

	u1	u2
x1	0	0
x2	0	0.003448
x3	-1	0
x4	0	-0.01695

The control input and the road excitation the state input is presented as:

$$u(t) = \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix} \tag{3.20}$$

C matrix is the output matrix that relates the state variables and the system output and it is given as:

C =

	x1	x2	x3	x4
y1	1	0	0	0
y2	0	1	0	0

(3.21)

y3	0	0	1	0
y4	0	0	0	1
y5	-57.97	-3.448	0	3.448

The output D matrix is a direct transmission matrix that connects the control inputs to the system output directly and it is calculated as;

D =

	u1	u2
y1	0	0
y2	0	0
y3	0	0
y4	0	0
y5	0	0.03448

The state vector $x(t)$ is given as:

$$x(t) = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} \tag{3.22}$$

Therefore, the state-space equation of a quarter car active suspension system model is obtained by substituting equation (3.17), (3.18), (3.19), (3.20), (3.21), (3.22) into equation (3.11), (3.12) respectively and it can be represented as follows:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{b_s}{m_{us}} & \frac{-(k_{us})}{m_{us}} & \frac{-(b_s+b_{us})}{m_s} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & \frac{1}{m_{us}} \\ -1 & 0 \\ \frac{b_{us}}{m_{us}} & \frac{k_s}{m_{us}} \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix}$$

By substituting the vehicle parameter values, the state space equation is described as:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.95 & 16.95 & -3220 & -16.949 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.003448 \\ -1 & 0 \\ 0 & -0.01695 \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix}$$

And the output equation $y(t)$ is given as:

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -57.97 & -3.448 & 0 & 3.448 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.03448 \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix} \tag{3.23}$$

3.8 Modelling quarter car passive suspension system

This modeling is done for comparison purpose to analyze the performance of the active suspension system with the passive system. The conventional suspension system contains the mass of the vehicle body which is connected with the mass of the wheel using spring and damper no actuator force is used that is why it is an uncontrolled type system. Figure 3.3 shows the modelling of active suspension system.

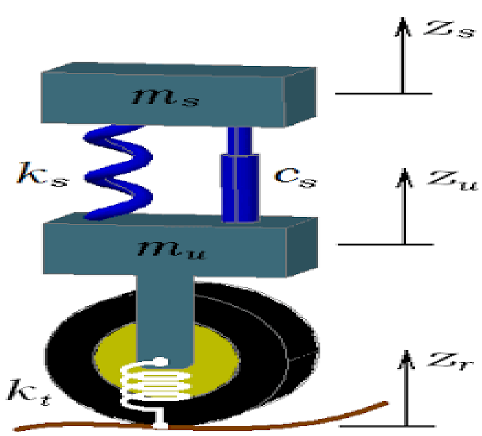

Figure 3.3 Passive suspension system model

Figures 3.4 shows free body diagram of 2 D.O. F passive quarter car suspension system.

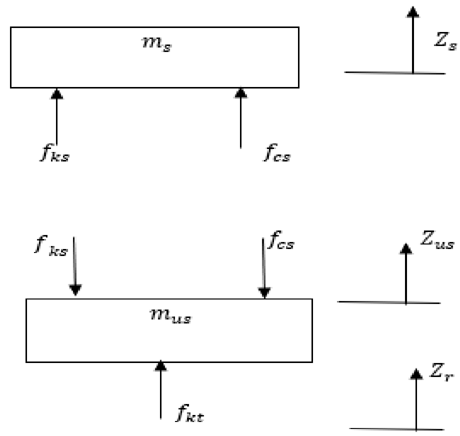

Figure 3.4 free body diagram for passive suspension system

From the above two figures mathematical equation that governs the system is derived. For the **sprung mass**; by applying newtons second law of motion, it is easy to find the net force acting on the sprung mass ($f_{net} = f_{ks} + f_{bs}$). Where the Net force (f_{net}) is equal to the sprung mass times the acceleration of the body.

$$m_s \frac{d^2 z_s}{dt^2} + k_s(z_s - z_{us}) + b_s \frac{d}{dt}(z_s - z_{u)} = 0$$

$$m_s \ddot{Z}_s = -b_s \dot{Z}_s + b_s \dot{Z}_{us} - k_s Z_s + k_s Z_{us} \qquad (3.24)$$

Dividing equation (3.15) by m_s we get:

$$\ddot{Z}_s = \frac{-k_s Z_s + k_s Z_{us} - b_s \dot{Z}_s + b_s \dot{Z}_{us}}{m_s} \qquad (3.25)$$

For the **un-sprung mass;**

$$f_{net} = f_{ks} + f_{bs} + f_{kt}$$

$$m_{us} \frac{d^2 z_{us}}{dt^2} + k_s(z_{us} - z_s) - k_t(z_r - z_{us}) + b_s \frac{d}{dt}(z_{us} - z_{s)} = 0$$

$$m_{us} \ddot{Z}_{us} = -k_s(Z_{us} - Z_s) + k_t(Z_r - Z_{us}) - b_s(\dot{Z}_{us} - \dot{Z}_s)$$

$$m_{us} \ddot{Z}_{us} = k_t Z_r - k_t Z_{us} + k_s Z_s - k_s Z_{us} + b_s \dot{Z}_s - b_s \dot{Z}_{us} \qquad (3.83)$$

Dividing equation (3.16) by m_{us} we get

$$\ddot{Z}_{us} = \frac{-(k_t+k_s)Z_{us}-b_s\dot{Z}_{us}+k_sZ_s+b_s\dot{Z}_s+k_tZ_r}{m_{us}}$$

(3.84)

Thus, the state space can be written as;

$x_1 = Z_s$, $\dot{x}_1 = \dot{Z}_s$

$x_2 = \dot{Z}_s$, $\dot{x}_2 = \ddot{Z}_s = \frac{-k_sZ_s+k_sZ_{us}-b_s\dot{Z}_s+b_s\dot{Z}_{us}}{m_s}$

$x_3 = Z_{us}$, $\dot{x}_3 = \dot{Z}_{us}$

$x_4 = \dot{Z}_{us}$, $\dot{x}_4 = \ddot{Z}_{us} = \frac{-(k_t+k_s)Z_{us}-b_s\dot{Z}_{us}+k_sZ_s+b_s\dot{Z}_s+k_tZ_r}{m_{us}}$

The state space matrix that describes relationship between input and output can be written as:

$$\dot{x}(t) = Ax(t) + Bu(t)$$

$$y(t) = Cx(t) + Du(t)$$

Substituting the given matrices, we get the state space equation of quarter car passive suspension system which is presented below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\frac{k_s}{m_s} & -\frac{b_s}{m_s} & 0 & \frac{b_s}{m_s} \\ 0 & 0 & 0 & 1 \\ \frac{k_s}{m_{us}} & \frac{b_s}{m_{us}} & \frac{-(k_t+k_s)}{m_{us}} & \frac{-(b_s)}{m_{us}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ \frac{-k_t}{m_{us}} \end{bmatrix} \begin{bmatrix} Z_r \\ F_a \end{bmatrix}$$

And the output equation $y(t)$ is given as:

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -57.97 & -3.448 & 0 & 3.448 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.03448 \end{bmatrix} \begin{bmatrix} Z_r \\ F_a \end{bmatrix} \quad (3.26)$$

Substituting the values of the vehicle parameters into equation (3.19) we get

$$\begin{bmatrix} \dot{x_1} \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & -1 \\ -57.97 & -3.448 & 0 & 3.448 \\ 0 & 0 & 0 & 1 \\ 284.95 & 16.95 & -3220 & -16.949 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0.003448 \\ -1 & 0 \\ 0 & -0.01695 \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix} \text{And}$$

the output equation $y(t)$ is given as:

$$y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -57.97 & -3.448 & 0 & 3.448 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0.03448 \end{bmatrix} \begin{bmatrix} \dot{Z}_r \\ F_a \end{bmatrix} \qquad (3.27)$$

In the case of passive suspension system, the actuator force is set to zero because there is no control input to the system the only input, we have is the road disturbance. The below figure shows Simulink model for quarter car passive suspension system.

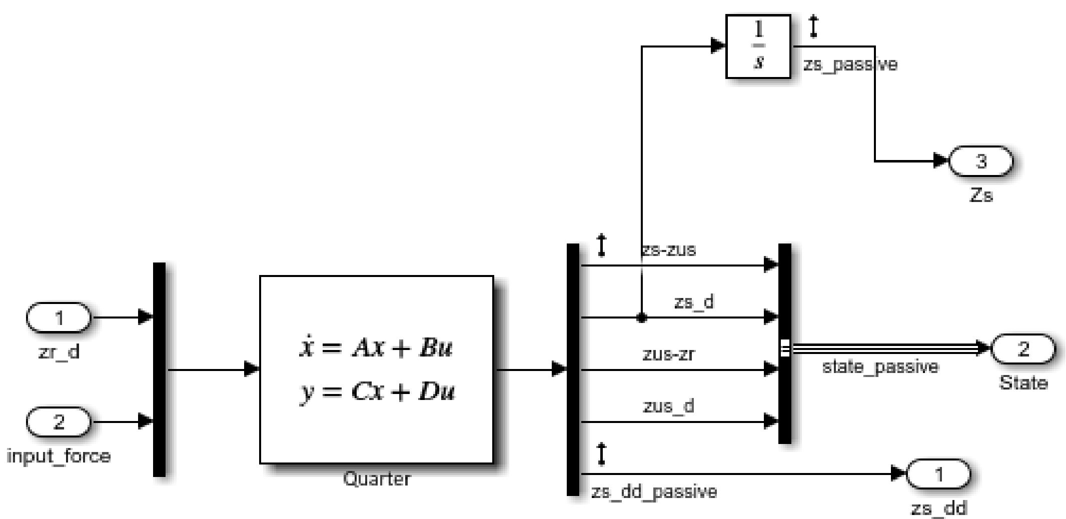

Figure 3.5 passive suspension system SIMULINK model

From figure 3.5 the system is modelled with two inputs but the input force in the case of passive suspension system control input is set to be zero. The state space equation is written in the standard form by the states pace block form the library of Simulink and the values of the parameters are inserted to obtain the outputs.

3.9 Controller Design

The proposed controllers are designed by the design procedures which is shown in figure 3.8. Firstly, a PID, LQR and Fuzzy-Logic controllers are designed. Then PID and Fuzzy-Logic controllers are tuned by using particle swarm optimization technique. Figure 3.6 clearly describes the procedures used in the design process.

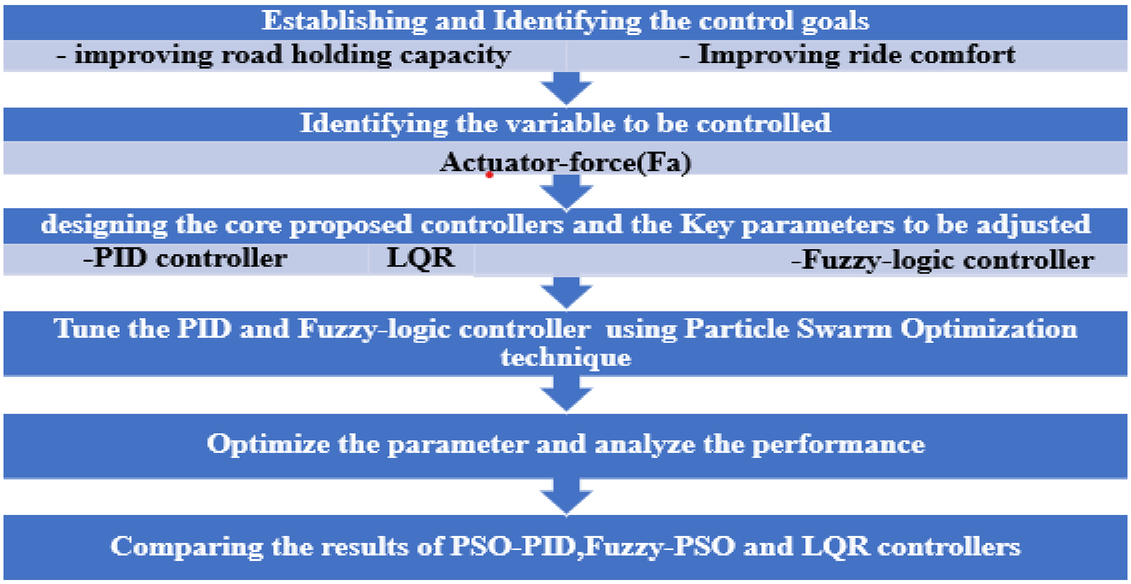

Figure 3.6 Control system design process

3.10 PID controller Design and implementation for Active suspension system

Industrial control applications have long made use of proportional integral and derivative (PID) controllers. PID controllers started with governor designs in the 1890s [20]. PID controllers are still used in most industrial applications even though they have been around for a while. Ninety percent of process industries employ them, according to a 1989 survey [21]. PIDs are widely used in industry since they are straightforward and simple to re-tune online. The ideas behind the generation and application of these phrases are depicted in the block diagram shown in the figure 3.7.

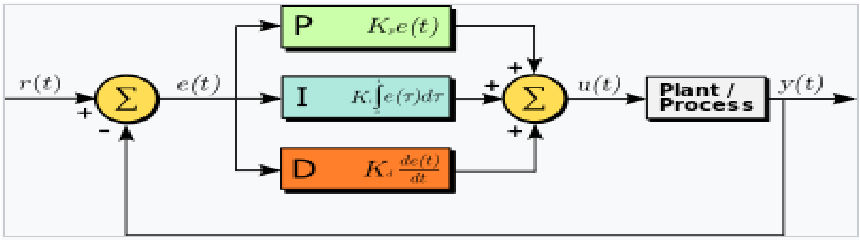

Figure 3.7 PID controller block diagram

Figure-3.9 shows the parameter proportions that are used. Derivative gain (Kd) improves stability and reduces overshoot, Integral gain (Ki) eliminates steady-state error but worsens transient response, and proportional gain (Kp) reduces rise time but does not eliminate steady-state error [22]. And equation 3.28 shows the general formula for the PID controller.

$$u(t) = k_p e(t) + k_i \cdot \int e(t)\, dt + k_d \frac{de}{dt} \tag{3.28}$$

In this research the PID controller the best values of the two PID gains. The design considered the above application of PID control and its way of implementation. As shown in figure 3.8 two PID controllers (PID_1 and PID_2) are used to generate and control the systems actuator force (F_a). The two PID controllers are designed for the sprung mass and un-sprung mass respectively. In the design process, two bump road profiles are selected as a reference input to the system as a velocity ($\dot{Z}_r$) which is the derivative of Z_r.

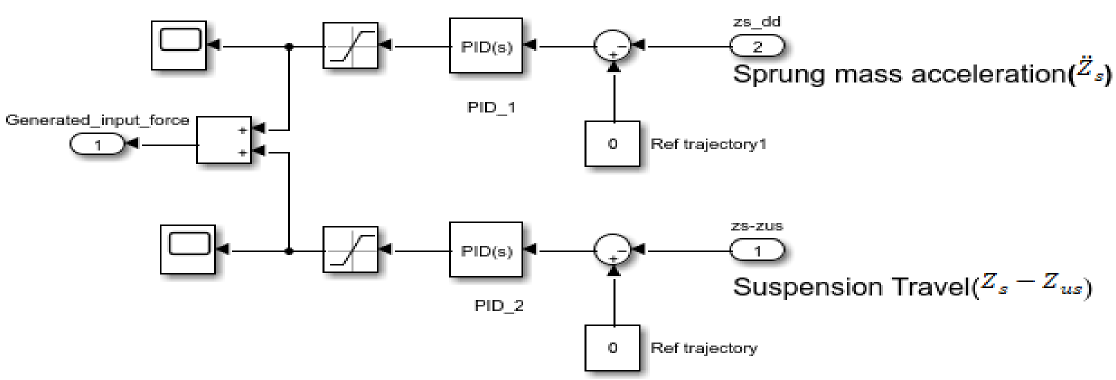

Figure 3.8 PID Controller Simulink design

The sprung mass acceleration ($\ddot{Z}_s$) and the suspension travel ($Z_s - Z_{us}$) are used as error signals with the summation of zero reference trajectory which are inputs for the two PID controllers. With the given inputs instead of trial-and-error PID tuner is used to tune the values of the gains (Kp_2, Ki_2, Kd_2) for PID_1 and (Kp, Ki and Kd) for PID_2. Euler and ode1 solvers are used with the given simulation time. The summation of the output generates the actuator force that minimizes the error signals which are sprung mass acceleration ($\ddot{Z}_s$) and the suspension travel ($Z_s - Z_{us}$) to have good ride quality. Figure 3.9 below shows the implementation of the designed PID controller for quarter car active suspension system. The desired closed loop dynamics is obtained by adjusting the two PID parameters (Kp_2, Ki_2, Kd_2, Kp, Ki and Kd), iteratively with" tuning" using PID tuner.

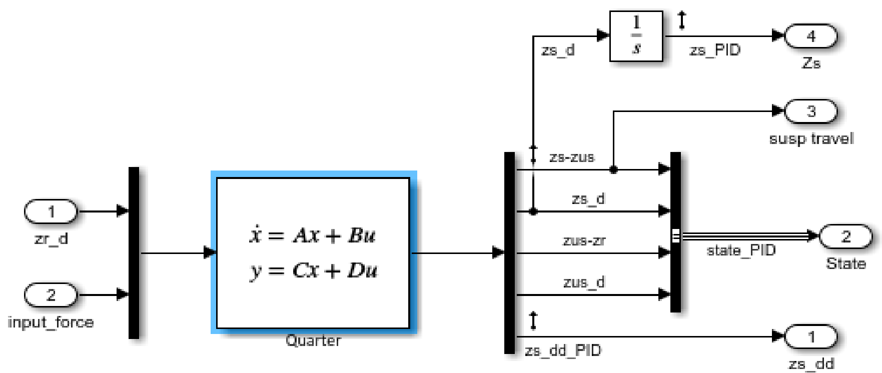

Figure 3.9 PID implementation for active suspension system

The generated actuator force that forms the closed loop dynamics is input to quarter car suspension system and used to reduce the tire deflection, the suspension travel, the un-sprung mass (displacement and velocity) and the un-sprung mass (displacement, velocity and acceleration) as shown in figure 3.9.

3.11 LQR Controller design and implementation for ASS

An effective method for figuring out the control gain, K, is provided by the LQR controller, a common kind of state feedback control. A tried-and-true control technique for linear Multiple-Input Multiple-Output (MIMO) time-invariant systems. Any LQR-based control system that hunts for the positive-definite solution to the algebraic Riccati problem which is represented in equation 3.28 below must satisfy the following conditions in order to minimize the performance index.

$$J = \frac{1}{2}\int_0^t (x^T Q x + U^T R U)\, dt \tag{3.28}$$

- The system must be completely state controllable
- R must be positive definite
- The system must be observable
- Q must be semi-positive definite

Where x^t is the state vector and contains the system state variable

U^t is the input vector and the system control input

It is necessary to alter both the weighting Q and R matrices. In order to lower the performance index or quadratic cost function J. After that, the weighting matrices are changed to achieve the necessary closed-loop performance. In order to penalize actuator effort or bad performance, the Q or R matrices are adjusted until the plant's cost function yields results that are satisfactory [23]. feedback regulator solution and performance index in complete state feedback controller, is shown in Figure 3.10 below.

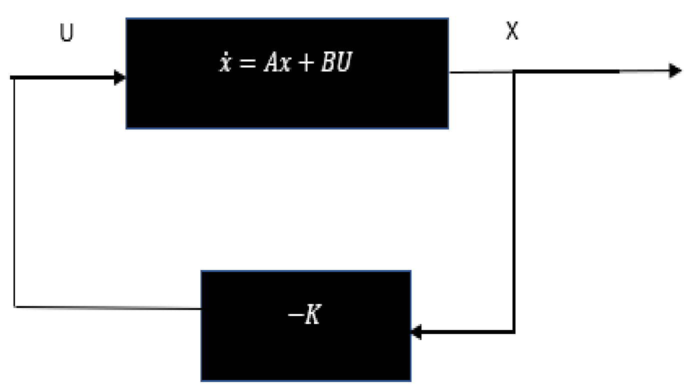

Figure 3.10 A full-state feedback block diagram

$$Fc = -Kx \tag{3.29}$$

In this paper, the linear quadratic regulator controller is applied to enhance vehicle handling by reducing the error on un-sprung mass reference to the ground, and ride comfort by minimizing the error between sprung and un-sprung mass. The goal of optimum control is to minimize the cost of function while simultaneously satisfying the physical restrictions. This involves determining a control vector actuator force and the behavior of the dynamic system to the required final state. In designing LQR both the A and B matrices must match to the actuator control force in the feedback regulator as mentioned in the mathematical model of quarter car suspension system. The LQR strategy will prioritize enhancing the road handling capability and riding comfort of the quarter car active suspension system road profile variant. As shown in equation (3.12) and (3.13), which describes the relationships between system control input U, system Y outputs, and x(t) state variables. In order for a linear time-invariant state system to be controllable, the controllability matrix P must have a complete rank and be equal to the number of states (n) in the system. There are two approaches used to identify whether a system is completely controllable or not, namely:

A) Kalman's method
B) Gilbert' method

Kalman's method will be explained because MATLAB implementation of controllability is based on this method.

$$P = [BABA^2B - - - A^{n-1}B]$$

$$rank(p) = n$$

Where:

P is controllability matrix

n is number of state variable

A is coefficient of matrix A

B is coefficient of matrix B

A system is controllable if and only if rank(p) = n. Whereas the controllability of the new model can be checked with the MATLAB command as:

rank (ctrb(A, B)) = 4

therefore rank(p) is equal to the number of the state variable. Therefore, the system is completely controllable. Now that the adjustable nature of the A and B matrices has been established and used to implement an LQR controller. MATLAB representations of the system matrix A, input matrix

B, output matrix C, and feedforward matrix D are initially required. Then the required Simulink model for LQR control designed as shown in figure 3.12 which will require the computed value of feedback gain value of K from MATLAB workspace during simulation. The controllability of the open-loop system has previously been defined therefore the weighting matrices Q and R of the quadratic performance index can be optimized and derived in MATLAB. The performance metrics are the most relevance and required more of a focus, which are multiplied to of the states connected to the suspension travel and the quarter car body acceleration. After trial and error of adjusting the non-zero entries in the Q matrix and the input weighting of the R matrix, the gain K value is obtained. Figure 3.11 is a simple way to implement LQR controller in MATLAB by using trial and error. By changing the Q and R matrices the best value gain K is obtained to enhance the ride comfort and road handling:

```
%% LQR Control law
 Q = diag([1.2*1e11, 1e9, 1, 1]);

%Q = diag([3.2*1e13, 1.01*1e11, -1e-8, -1]);
%Q = diag([1e13, 1e11, 1e6, 1e5]);
%Q = diag([11.31e12, 4*1e9, 8*1e4, 1e-20]);
%Q = diag([1e13, 1e11, 1e6, 0.001]);
%Q = diag([1e13, 1e11, 1e8, 1e-11]);

R = 1;
 %Q = diag([1760*10^6, 11.6*10^6, 1, 1]);
 %R = 0.01;

% feedback gain vector
K = lqr( A, B(:,2), Q, R );
```

Figure 3.11 LQR design using MATLAB code

Figure 3.12 shows the Simulink design of LQR that shows how the obtained gain K is going to generate the desired actuator force (Fa) that is used to minimize the tire deflection, suspension travel, un-sprung mass velocity, sprung mass displacement and sprung mass acceleration to have good ride quality.

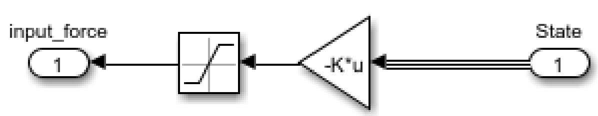

Figure 3.12 linear quadratic regulator Control

Figure 3.13 shows implementation of the above designed LQR controller for the active suspension system. As shown in the figure there are two input which are the control input (the actuator force) generated by the LQR, which is input to the suspension system and reference input (road profile). The designed gain of the LQR is used to multiply the control input for better performance and road holding capacity by the generated actuator force. The output of the system is stated clearly in the figure and the result is discussed on the next part of the paper.

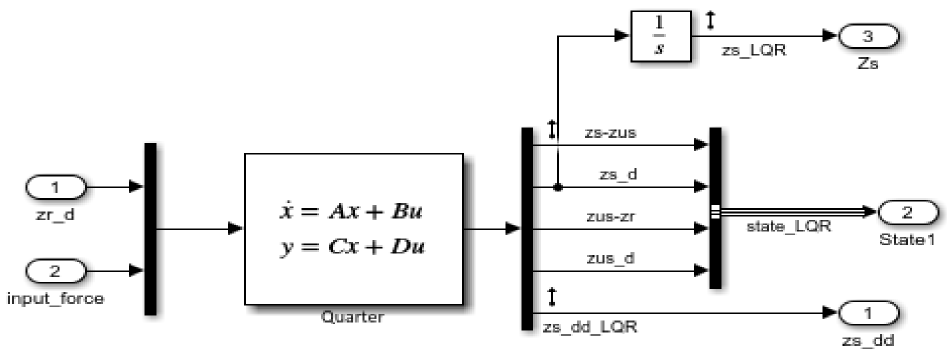

Figure 3.13 LQR implementation for quarter car active suspension system

3.12 Fuzzy logic controller Design

Fuzzy logic control is an artificial intelligence technique that mimics human thinking to regulate complex systems. Unlike typical control systems that rely on precise mathematical models, fuzzy logic employs terms like "low," "medium," and "high" to express system states. Dealing with the ambiguity and uncertainty that accompany real-world situations is made feasible by this strategy. Systems can be successfully managed using fuzzy logic controllers without a deep understanding of their internal operations. [24] They achieve this by de-fuzzifying the fuzzy output back into numerical values after first converting numerical input into fuzzy words (fuzzification), applying rules that approximate human rules, and so forth. Nonlinear and unpredictable systems can be effectively managed using fuzzy logic control, however because it necessitates rule-based design and specialized knowledge, it can be difficult and expensive to implement.

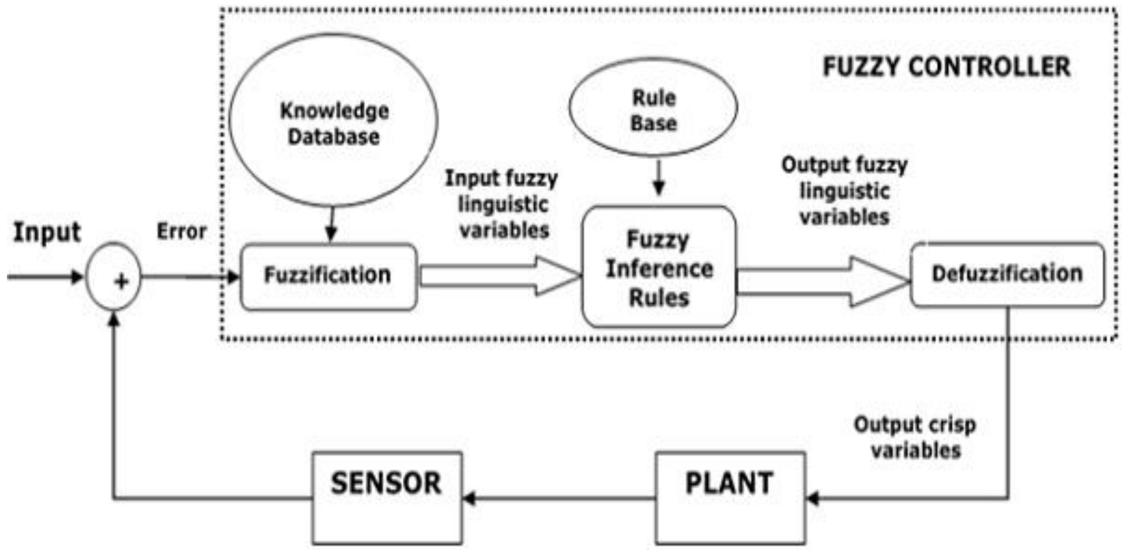

Figure 3.14 fuzzy logic controller design procedure

In this research paper the fuzzy logic controller is designed by taking the procedure from figure 3.14 which consists of following stages.

1. **Input variable stage**: In the design the two crisp inputs are defined as suspension deflection (error) and the body velocity (error rate).
2. **Membership Functions:** In this fuzzification stage membership function is constructed for the two crisp inputs by a shape of triangular membership plot which describes how each input value belongs to different linguistic terms or fuzzy sets.

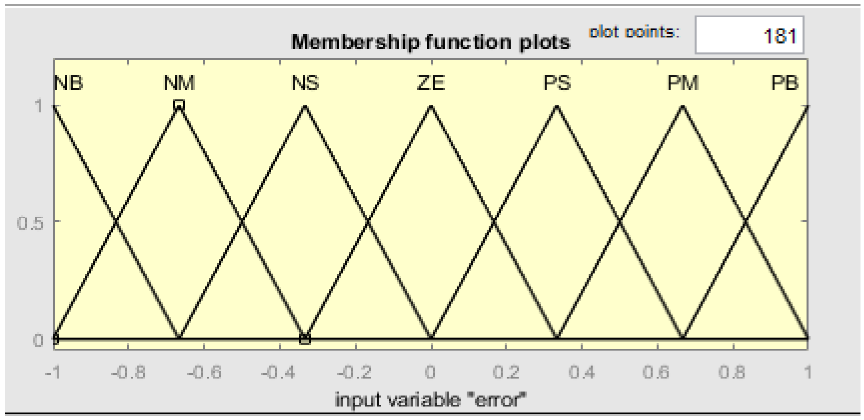

Figure 3.15 Membership function of suspension deflection

From the above figure we can observe that the membership function for the suspension deflection is constructed with a range of [-1 1]. The same range is also applied for the body velocity membership function as shown in the figure below.

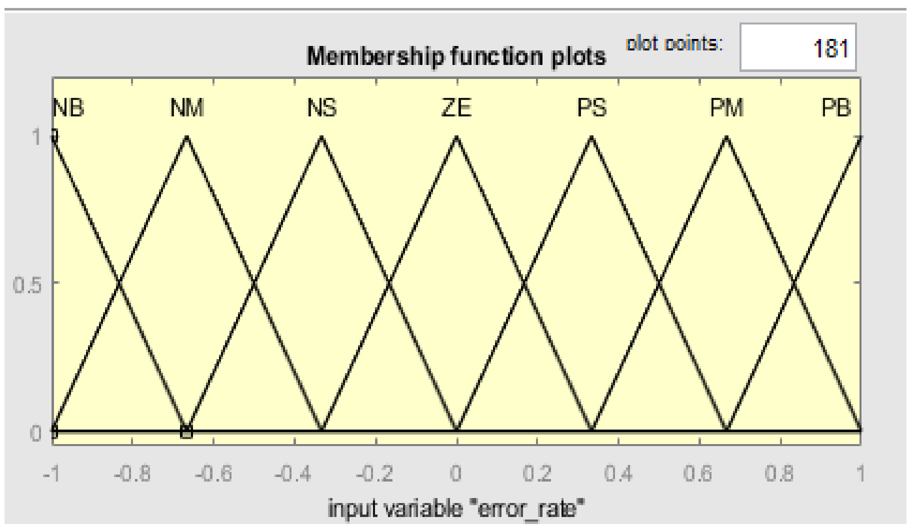

Figure 3.16 Membership function of the body velocity

3. **Fuzzy Linguistic Terms:** In constructing the member ship functions for the input variables seven linguistic terms are used which are negatively-big *(NB)*, Negatively Medium *(NM)*, Negatively-Small (NS), Zero *(ZE)*, Positively-Small *(PS)*, Positively-Medium *(PM)* and Positively-Big *(PB)*. The Fuzzy Membership Degree is calculated as the degree to which a two crisp input values belong to each of the fuzzy sets defined by the membership functions. The fuzzy membership degree and the fuzzy set of the two inputs variables for each member are described in table 2.

Table 2 Input membership function values

`membership functions	Values
NB	$[-1 \quad -1 \quad -0.666666666666667]$
NM	$[-1 \; -0.666666666666667 \; -0.333333333333333]$
NS	$[-0.666666666666667 \; -0.333333333333333 \quad 0]$
ZE	$[-0.333333333333333 \quad 0 \quad 0.333333333333333]$
PS	$[0 \quad 0.333333333333333 \quad 0.666666666666667]$
PS	$[0.333333333333333 \; 0.666666666666667 \; 1]$
PB	$[0.666666666666667 \quad 1 \quad 1]$

4. **fuzzy Inference System (FIS):** In this stage the inference engine is used to create rule-based systems by a collection of fuzzy IF-THEN rules that emulate human decision-making processes and the fuzzy rules are applied to the fuzzy input data to generate fuzzy output and the rules that connect input fuzzy sets to output fuzzy sets are defined. For this design Mamdani approach is used to create the fuzzy inference system. The number of the membership function are $n = 7$ such that there are $n^2 = 49$ rules that connect input fuzzy sets to output fuzzy sets. The table below is constructed by IF-THEN rule for example if we take row 2 and column 1 it indicates ‘If (error is PB) and (error rate is PB) then (actuator force is PB)’ like this all are stated in table 3.

Table 3 Rule Base in the FLC design

ΔE / E	PB	PM	PS	ZE	NS	NM	NB
PB	PB	PB	PB	PB	PM	PS	ZE
PM	PB	PB	PB	PM	PS	ZE	NS
PS	PB	PB	PM	PS	ZE	NS	NM
ZE	PB	PM	PS	ZE	NS	NM	NB
0NS	PM	PS	ZE	NS	NM	NB	NB
NM	PS	ZE	NS	NM	NB	NB	NB
NP	ZE	NS	NM	NB	NB	NB	NB

5. **Output Membership Function**: combine the outputs of different rules to obtain a single aggregated fuzzy set that represents the system's response. The system has one output

variable which is the actuator force and the membership function which ranges [-1 -1] and like the inputs the output have seven triangular shaped membership functions that describes the action of the actuator force. And each membership function has a fuzzy set that is stated in table 4.

Table 4 Output membership function values

`membership functions	Values
NB	$[-1\ \ -1-0.75]$
NM	$[-0.85\ \ -0.6\ \ -0.35]$
NS	$[-0.5\ \ -0.25\ \ 0]$
ZE	$[-0.25\ \ 0\ \ 0.25]$
PS	$[0\quad 0.25\quad 0.5]$
PS	$[0.35\ 0.6\ 0.85]$
PB	$[0.75\quad 1\quad 1]$

Figure 3.17 describes the output membership function from the range of [-1 1] and the membership functions are plotted based on the value described in table 4.

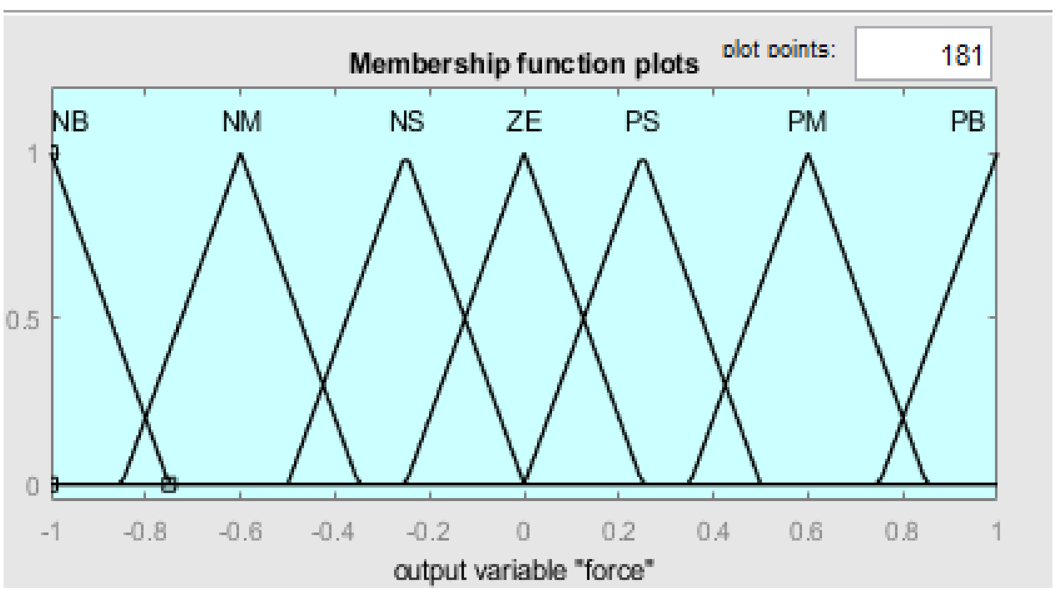

Figure 3.17 Actuator force membership functions

With this fuzzy set the membership functions and the surface area where the action of the actuator force is generated is described in the figure 3.18 below.

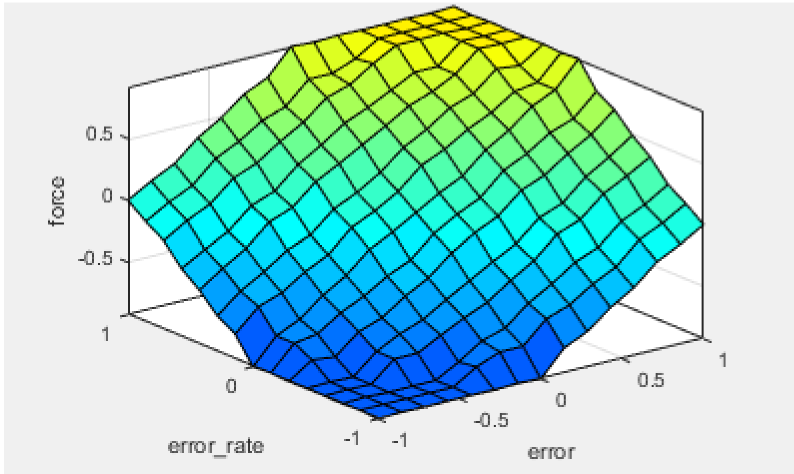

Figure 3.18 The output surface of the fuzzy system

In the next design we are going to tune the fuzzy logic controller using Particle Swarm optimization and we are going to find the tuned value of the inputs and the output within the range that minimizes the tire deflection, suspension travel, body acceleration and the un-sprung mass velocity.

3.13 Particle swarm optimization (PSO)

as Drs. Kennedy and Eberhart have said the revolutionary optimization technique known as population-based optimization (PSO) was inspired by flocking birds and schooling fish. Unlike genetic algorithms, PSO avoids complex operators like crossover and mutation. Instead, it relies on particles (potential solutions) that travel around the problem space, guided by the past achievements of both the individual particles and the collective swarm. PSO's ease of use and rapid convergence to effective solutions are contributing to its growing popularity.[25].

3.14 PSO tuned PID and fuzzy logic controller design

In this design PSO is used to tune the two PID gains and the fuzzy logic parameters that are going to multiply the error, the derivative of the error and the output of the fuzzy system. The two PID controllers are designed using block configuration using gain block from the Simulink library the six parameters which are (Kp_2, Ki_2, Kd_2, Ki, Kp and Kd) are configured and using PSO technique the optimal value of the parameters(Kp_2, Ki_2, Kd_2, Ki, Kp and Kd) used in Simulink model that minimizes an objective function to avoid trial and error problems due to guessing the value of the gains. Figure 3.19 shows two PID blocks and the PSO objective function sprung mass acceleration and suspension travel (F2) that has to be minimized

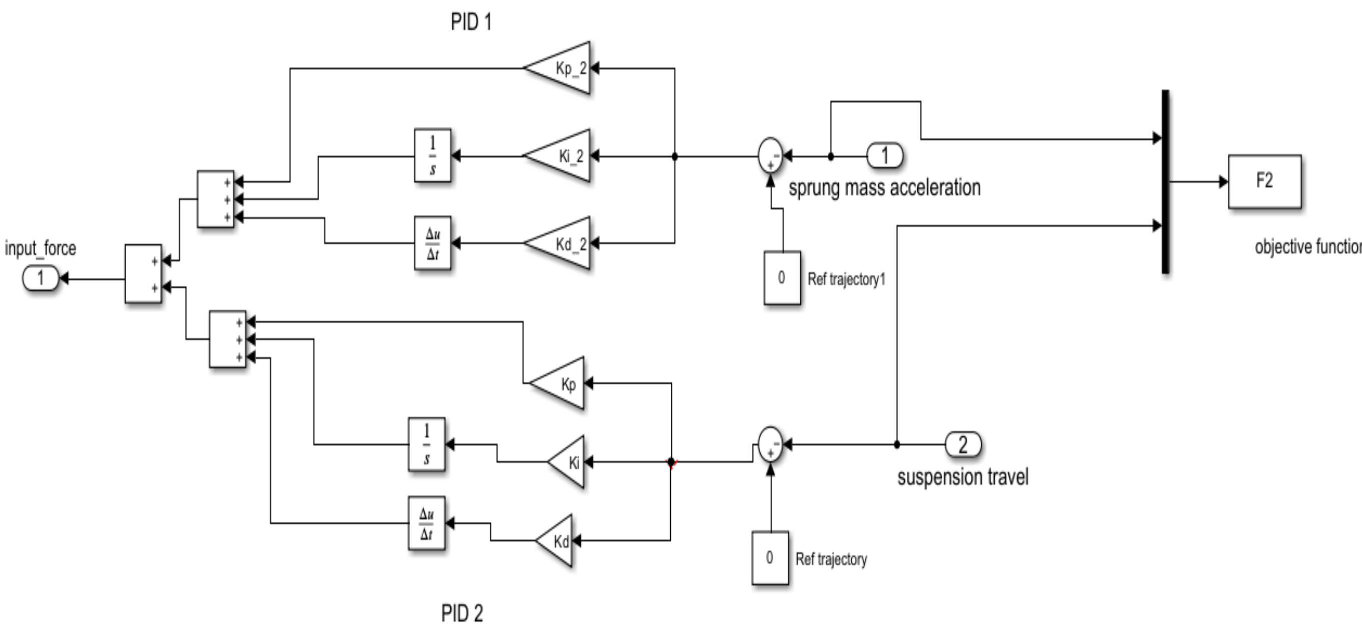

Figure 3.19 Two PID controllers by block configuration

For the fuzzy controller three parameters the Particle Swarm Optimization (PSO) algorithm is implemented to solve a three-dimensional optimization problem. It searches for optimal values of parameters (P, S, and O) used in a Simulink model that minimizes an objective function(F) in this case the sprung mass displacement is taken as an objective function. To avoid unwanted throwing errors by fuzzy due to inputs and output bounded value PSO can settle this error by applying the fitness function which can determine the range for both inputs and output by this approach the performance of fuzzy will improve. Figure 3.20 shows the Simulink design of FUZZY-PSO for the active suspension system.

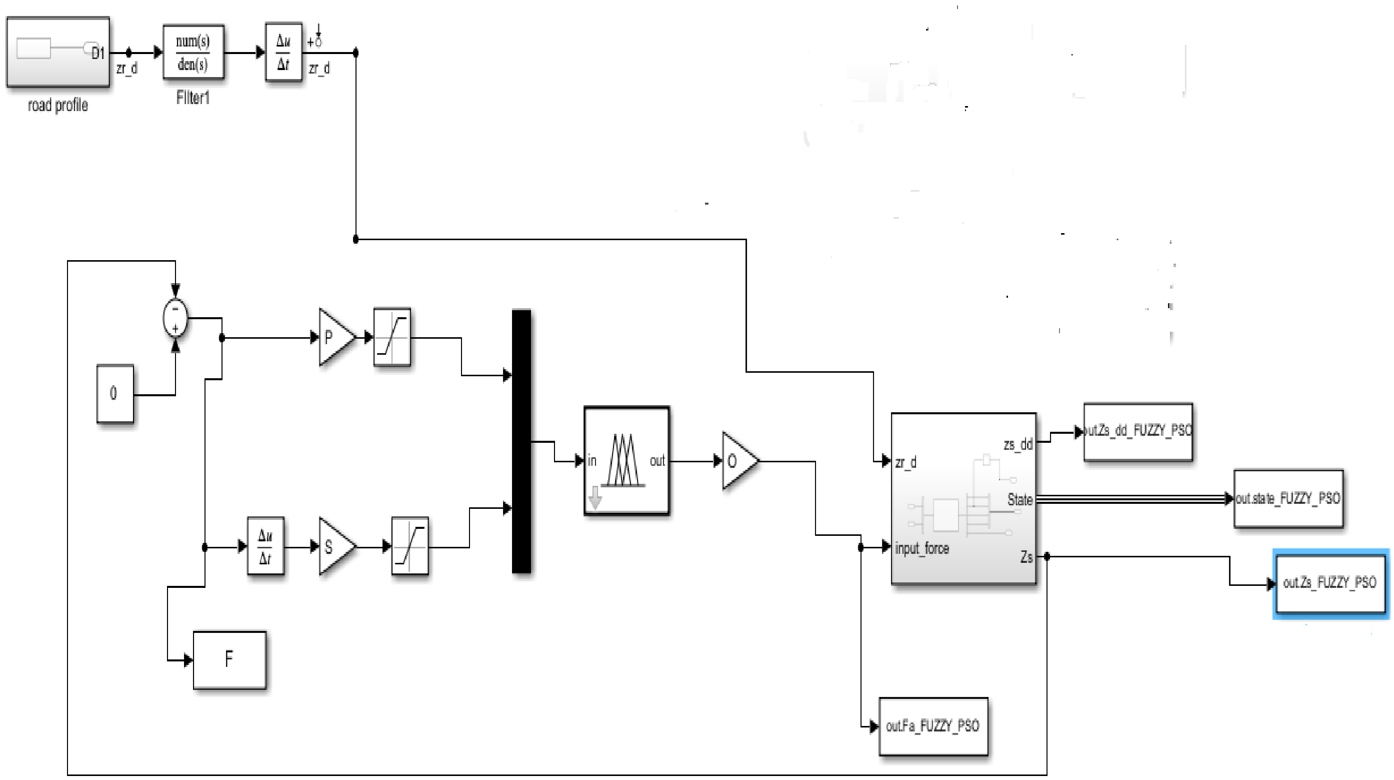

Figure 3.20 FUZZY-PSO implementation on quarter car active suspension system

The simulation result of this controller and the comparative analysis of the two-suspension system are discussed in the next section of this paper. Figure 3.21 shows the algorithm used for PSO to tune PID and Fuzzy logic controller.

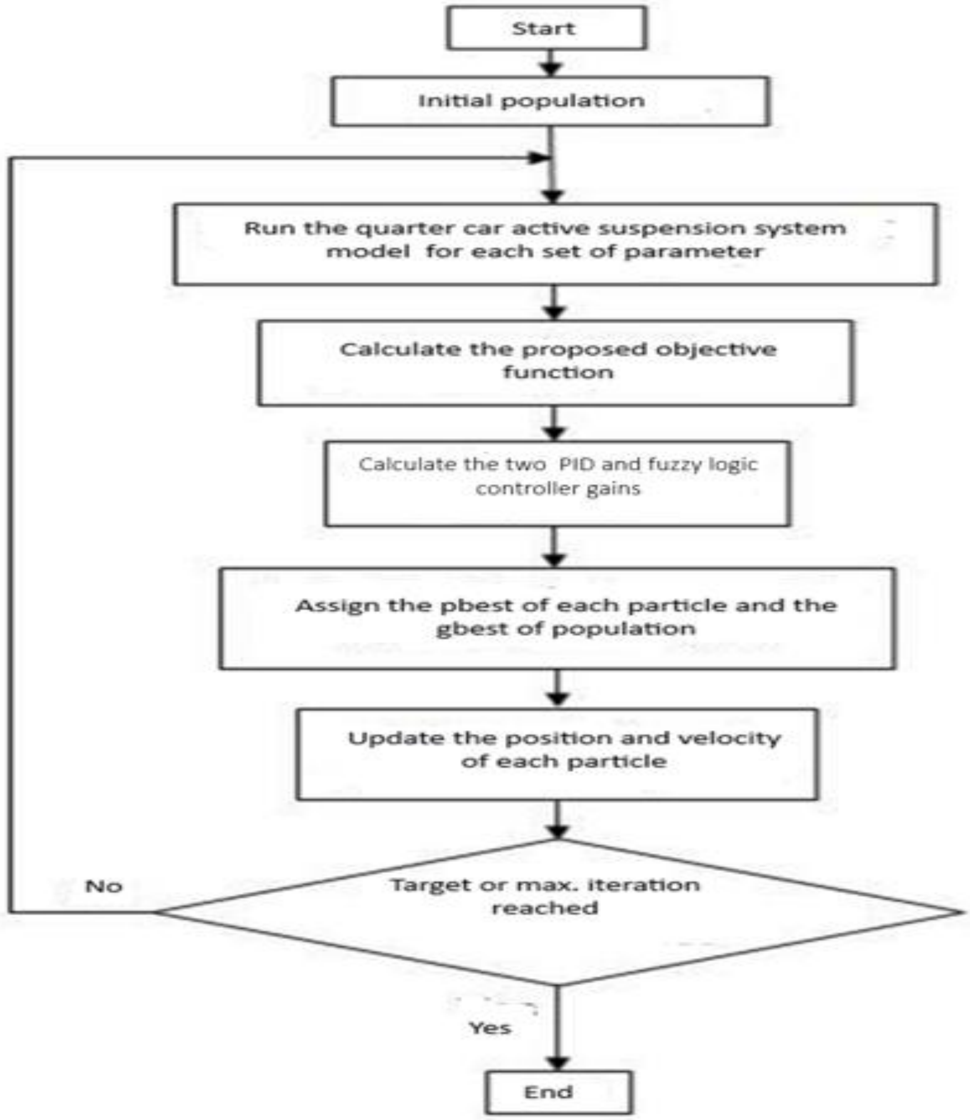

Figure 3.21 PSO tuned fuzzy and PID controller’s algorithm

The PSO algorithm maintains a population of particles, each of which represents a potential solution, as seen in figure 3.21. Iteratively updating their positions based on the best places they have located thus far (Particle Best) and the best position the entire swarm has found (Global Best), particles scan the search space. Numerous PSO parameters, including population size, maximum number of iterations, and weighting variables, may be adjusted according to the design. Following the search, the algorithm displays the optimal values for each of the three variables as well as the solution with the highest fitness (objective function value). Additionally, it shows the convergence behavior, which illustrates how the optimal fitness increases with repetitions.

These are the main equations:

Particle Position: PID and fuzzy controller parameters are represented by each particle. For particle i, these locations are represented as X_i.

Particle Velocity: Every particle's motion inside the search space is dictated by its velocity, or V_i. The update is based on the global best position (gbest) that has been identified thus far, as well as its own best position, personal best (pbest).

Velocity update:

$$V\_i\,(t+1) = w * V\_i\,(t) + c\_1 \; * rand(\,) * \,(P\_(best\,i)\,(t) - X\_i\;(t)) + c\_2 \; * rand(\,) * \,(G\_(best\,i)\,(t) - X\_i\;(t))$$

Position update:

$$X\_i\;(t+1) \;=\; X\_i\;(t) + V\_i\;(t+1)$$

Particle locations are iteratively updated by these PSO formulae, which push the particles in the direction of the fuzzy controller and PID controller settings that optimize the selected performance metric that is, minimizing suspension travel and sprung mass acceleration. The research paper used the following PSO parameters presented in table 5.

Table 5 parameter values for particle swarm optimization

PSO parameters	Values
M	9
N	20
W	0.9
c_1	1.2
c_2	1.4
max_iterations	10
max_no_runs	1
Lower Limit	[90 115 1000 200 115 1000 0 0 1500]
Upper Limit	[400 400 4000 800 200 4000 15 15 2100]

where:

- m - number of variables
- w - inertia weight (controls momentum)
- c_1 - cognitive learning rate (importance of pbest)
- c_2 - social learning rate (importance of gbest)
- Rand () - random number between 0 and 1

CHAPTER FOUR

4. RESULT AND DISCUSSION

In chapter three, the mathematical model of both the passive and active suspension systems are derived and each equation is represented in state-space form for a clear understanding and to make the study easy. To demonstrate the efficacy of the suggested techniques, numerical simulation results are presented in this chapter.

4.1 Parameter tuning of PID and LQR controllers

The two controllers have different control variables: the LQR controller uses the feedback regulator to control all state space variables, whereas the PID controller just uses the suspension travel and sprung mass acceleration. For the two controllers, a "trial and error" approach, a series of simulations have been conducted to determine the parameters that maximize the performance parameters for the PID controller. The MATLAB function “pidtune” has been used, which given the transfer function and the type of controller (PID in this case) returns the values of the parameters (KP_1, Ki_2, Kd_2, Kp, Ki, and Kd). The parameters that were acquired are detailed in the table as follows:

Table 6 Parameter tuning result for PID 1 controller

Control variable	Kp_2	Ki_2	Kd_2
$\ddot{Z}_s$	160	7.369267450956900e+03	0

Table 7 Parameter tuning result for PID 2 controller

Control variable	Kp	Ki	Kd
zs_zus	6.9205e+04	1.7945e+05	5.5275e+03

A number of simulations have run to select the right weighting matrices Q and R for the LQR controller. Q is a diagonal positive definite matrix, while R is a positive constant. The process starts by setting R to 1 and assigning values to the diagonal elements of Q. The MATLAB "lqr" function then computes the gain matrix K based on Q, R, and the state space matrices A and B. Next, the PID controller helps to adjust the parameters through a "trial and error" method to reduce the quadratic cost function J. The chosen parameters are:

$$Q = \begin{bmatrix} 1.75e+13 & 0 & 0 & 0 \\ 0 & 1.02e+11 & 0 & 0 \\ 0 & 0 & -1e-08 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \text{ and } R = 1 \tag{4.1}$$

$K = [4.1665e + 06, 3.2204e + 05, 4.7847e + 03, -2.66387e + 03]$

4.2 Performance analysis of PSS and ASS with PID and LQR with two Bump road input

The response of the system is tested considering a speed bump profile, which corresponds to Figure 4.1. The speed bump profile has been modelled using the following parameter described in table 9and equations:

Table 8 parameters for the random road profile

Parameters	Values in (m)
Bump_height1 (h_1)	0.02
Bump_height2 (h_2)	0.04
Bump_width1 (w_1)	0.6
Bump_width2(w_2)	0.8
Bump1_center (c_1))	1.5
Bump2_center (c_2)	3.0

The speed bump profile has been modelled using the following equations described below:

$$\begin{cases} Zr_1 = h_2 * (1 - (\cos 2\pi(o.5t - (0.5\frac{c_1}{w_1} - 1))) \;, & 1.2 \le t \le 1.8 \\ Zr_2 = h_2 * (1 - \cos(2\pi(0.5t - (0.5\frac{c_2}{w_2} - 1))) \;, & 2.2 \le t \le 3.4 \\ 0 & otherwise \end{cases} \tag{4.2}$$

Where the variables t_i and t_f are defined depending on the vehicle velocity. In this paper the simulation has been done considering the average velocity, (80 km/h → t_i = 1.2 s and t_f = 3.4s. figure 4.1 shows the speed bump profile for a vehicle moving at 80 km/h.

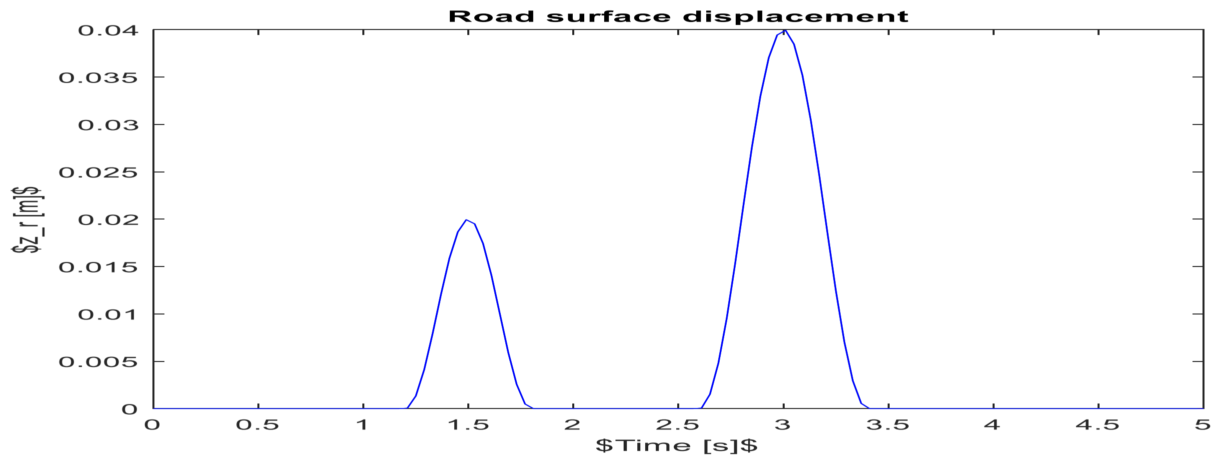

Figure 4.1 Road surface displacement

Figure 4.1 shows the result of the selected road profile as an input to the system with this road input the following results are obtained and active suspension system with passive suspension system are compared. For the active suspension system LQR and PID controller with PID tuner are used and compared.

to evaluate the performance of the active suspension system with respect to the passive ones two different performance indices have been defined, equation 4.3 is used to evaluate the percentage reduction of the oscillations and equation 4.4 is used to evaluate the percentage reduction of the overshoot, Equation

$$\left(KPI_{RMS} = 1 - \frac{\mathrm{RMS}(|X_{PID,\ LQR}|)}{\mathrm{RMS}(|X_{passive}|)}\right)\% \tag{4.3}$$

$$\left(KPI_{MAX} = 1 - \frac{MAX(|X_{PID,\ LQR}|)}{MAX(|X_{passive}|)}\right)\% \tag{4.4}$$

4.2.1 Tire deflection for active suspension system and passive suspension system

Figure 4.2 shows comparative analysis of the tire deflection response of the PSS and ASS.

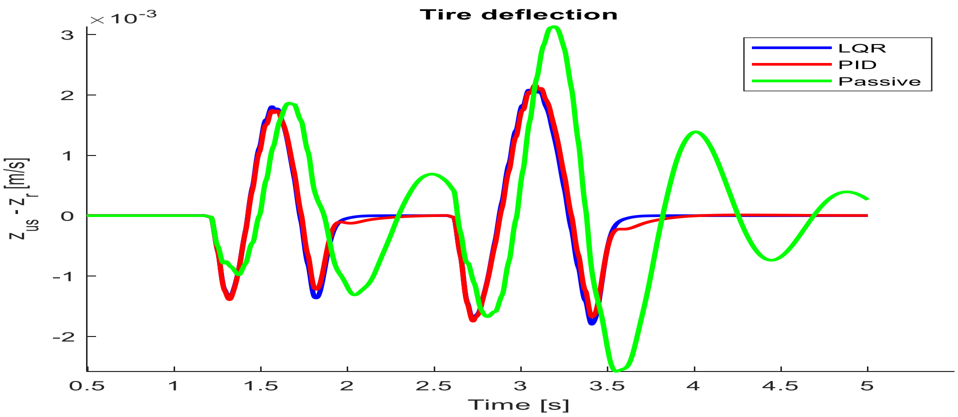

Figure 4.2 Tire deflection for passive and active (with PID and LQR) controllers

Observing figure 4.2 the PID and LQR controllers reduce the peak value from in the positive peak direction based on the two PID gains and the LQR gain. The LQR shows a higher reduction of the tire deflection compared to the PID. Whereas in the negative peak direction, the PID controller shows a higher value reduction compared to LQR by giving a response to the change of the suspension system based on the generated force. whereas the LQR controller shows the lower value reduction in the negative peak direction by giving a response to the change of the suspension system based on the generated force by two PID gains. The table below compares the tire deflection of passive and active (using PID and LQR) suspension system to the positive peak direction.

Table 9 $\mathrm{KPI_{MAX}}$ for ASS (PID and LQR controllers) and PSS on tire deflection response

Parameter	MAX - passive	MAX- PID controller	MAX- LQR controller	($\mathrm{KPI_{MAX}}$) PID	($\mathrm{KPI_{MAX}}$) LQR
Tire deflection ($zus - zr$) (m)	0.0031	0.0022	0.0021	29.03%	32.25%

From table 9 taking the maximum values from the positive direction an active suspension system with LQR controller has decreased the tire deflection overshoot by 0.001m which is greater than the PID controller in reducing the overshoot of the tire deflection by 0.0009 m when comparing with the uncontrolled passive suspension system. As mentioned in table 9 an active suspension system with LQR and PID controllers has good reduction in tire deflection from the positive peak direction. Table 10 shows the oscillation reduction of the two controllers comparing with the passive suspension system.

Table 10 KPI_{RMS} for ASS (PID and LQR controllers) on tire deflection response

Parameter	RMS-passive	RMS-PID controller	RMS-LQR controller	(KPI_{RMS}) PID	(KPI_{RMS}) LQR
Tire deflection $(zus - zr)$ (m)	0.0011	$6.8572*10^{-4}$	$6.9373*10^{-4}$	37.66%	36.9%

The root mean square of PID controller in reduction of oscillation of the tire deflection is $6.8572 * 10^{-4}m$ this tells us from the tire deflection of passive suspension system oscillation is reduced by 0.0004143m and using LQR controller oscillation of the tire deflection reduction is $6.9373 * 10^{-4}$ m as so that LQR controlled active suspension system decreased the tire deflection by 0.000406m when comparing with passive suspension system as shown in table 10.

4.2.2 Suspension travel for active and Passive suspension system

The figure depicts comparative analysis of the suspension travel response of the passive and the active suspension system.

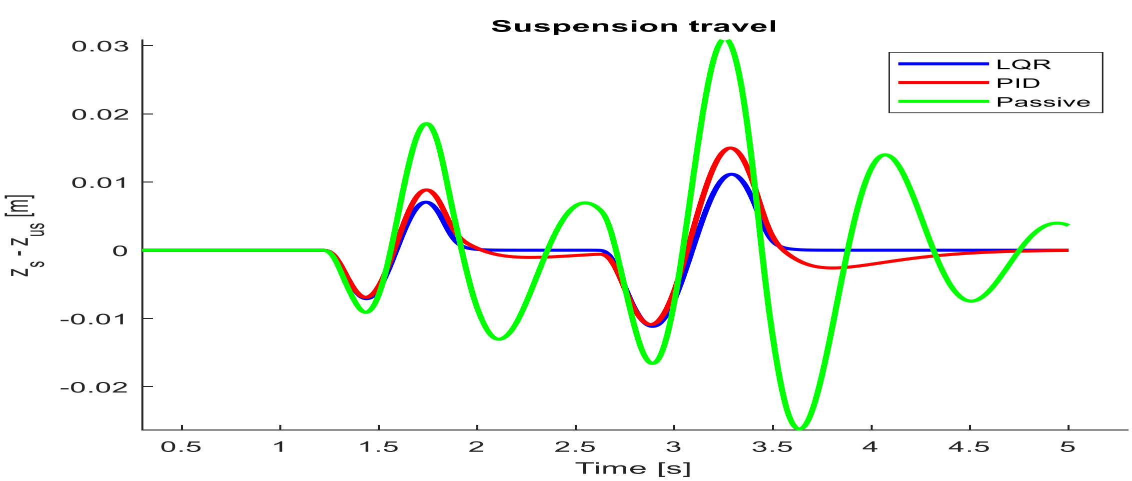

Figure 4.3 Suspension travel for passive and active (with PID and LQR) controllers

The peak-to-peak value comparison of the LQR controller reduces the peak value from in the positive peak direction compared to PID controller. Whereas in the negative peak value direction, both the LQR and the PID controller shows a higher value reduction compared to passive by giving a response to the change of the suspension system based on the force generated by feedback gain value of K .and the two PID gains. The tables below show the key performance index of ASS controlled using PID and LQR controllers in reduction to the suspension travel.

Table 11 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on suspension travel response

Parameters	MAX - passive	MAX- PID controller	MAX- LQR controller	$(\mathrm{KPI}_{\mathrm{MAX}})$ PID	$(\mathrm{KPI}_{\mathrm{MAX}})$ LQR
Suspension travel (zs_zus) (m)	0.0310	0.0150	0.0111	51.6%	64.2%

From table 11 LQR and PID have reduced the overshoot of the systems suspension travel by 0.0111 m and 0.0150 m respectively this indicates an ASS has decreased the overshoot caused by the road bump by more than half percent so that the designed controllers can give better road handling performance compared to the passive suspension system. Table 12 shows is the performance analysis of the two controllers in reduction of the suspension travel oscillation.

Table 12 KPI_{RMS} for ASS (PID and LQR controllers) and PSS on suspension travel response

Parameter	RMS- passive	RMS-PID controller	RMS-LQR controller	$(\mathrm{KPI}_{\mathrm{RMS}})$ PID	$(\mathrm{KPI}_{\mathrm{RMS}})$ LQR
Suspension travel $(zus - zr)$ (m)	0.0105	0.0044	0.0036	58.09%	65.7 %

From table 12 LQR and PID have reduced the oscillation of the systems suspension travel to 0.0036 m and 0.0044 m respectively when comparing with the uncontrolled suspension system. which means both the LQR an PID controlled ASS has good road holding capacity compared to the PSS.

4.2.3 Un- sprung mass velocity for active and Passive suspension system

Figure 4.4 shows the un-sprung mass velocity of a suspension system over time. The three curves represent the velocity of the un-sprung mass for passive and active suspension system which is controlled by PID and LQR.

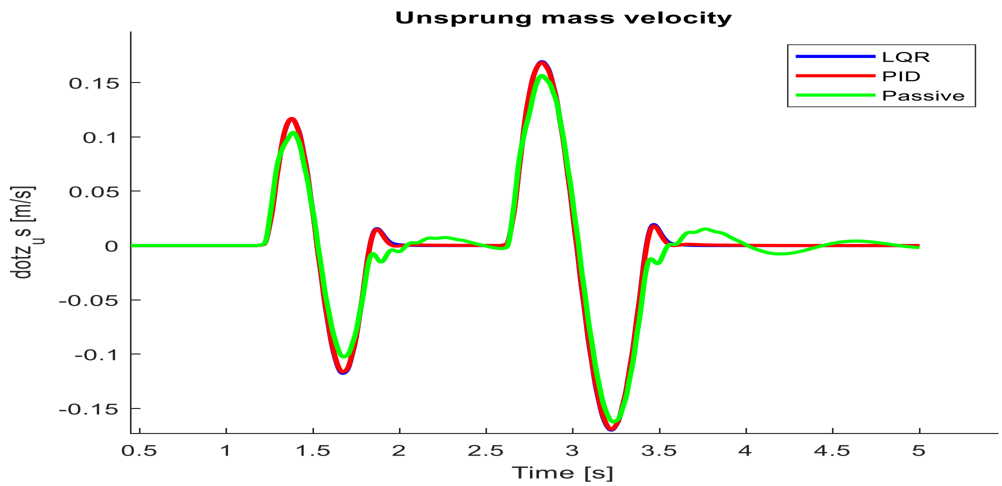

Figure 4.4 Un-sprung mass velocity for passive and active (with PID and LQR) controllers

The graph shows that the two suspension systems experience periods of positive and negative velocity, which means the un-sprung mass is moving up and down. However, the magnitude of the velocity is negative for the PID and LQR controlled active suspension system compared to the Passive suspension system. Table 13 shows the peak-to-peak value comparison of the PID and LQR controllers' reduction of the overshoot of the system.

Table 13 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on un-sprung mass velocity response

Parameters	MAX - passive	MAX- PID controller	MAX- LQR controller	(KPI_{MAX}) PID	(KPI_{MAX}) LQR
Un-sprung mass velocity ($z\dot{u}s$) (m/s)	0.1565	0.1684	0.1692	-7.6%	-8.1%

Table 13 shows an active suspension system with PID and LQR controllers have negative reduction overshoot as well as negative oscillation of the velocity of the un-sprung mass with two bump input road profile in table 14.

Table 14 KPI_{RMS} for ASS (PID and LQR controllers) and PSS on un-sprung mass velocity response

Parameter	RMS- passive	RMS-PID controller	RMS-LQR controller	(KPI_{RMS}) PID	(KPI_{RMS}) LQR
Un-sprung mass velocity ($z\dot{u}s$) (m/s)	0.0522	0.0551	0.0553	-5.5%	-5.9%

4.2.4 Sprung mass displacement for active and Passive suspension system

Figure 4.5 shows the sprung mass displacement of a quarter car active and passive suspension system over time.

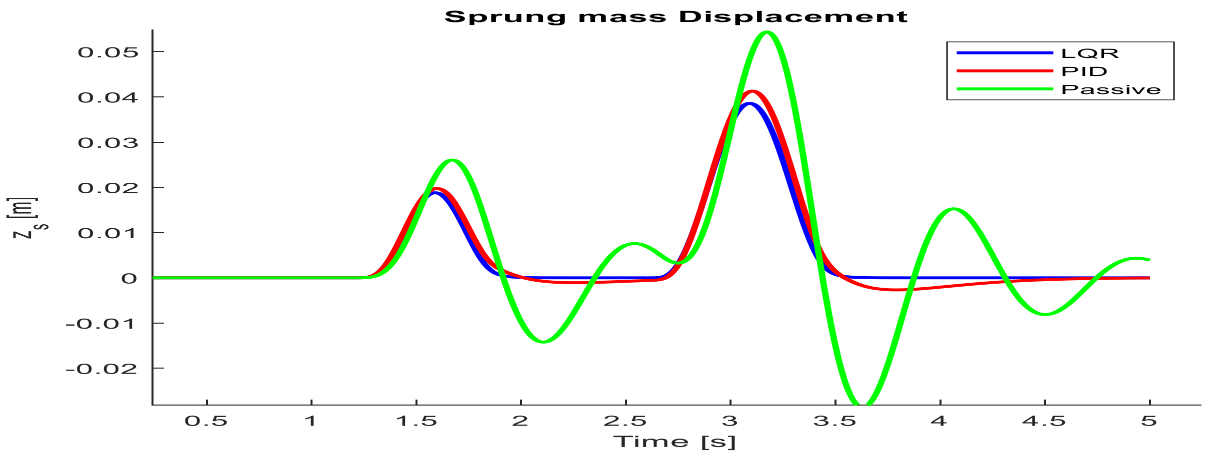

Figure 4.5 sprung mass displacement for passive and active (with PID and LQR) controllers

Table 15 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on sprung mass displacement response

Parameter	MAX - passive	MAX- PID controller	MAX- LQR controller	(KPI_{MAX}) PID	(KPI_{MAX}) LQR
Sprung mass displacement (zs) (m)	0.0543	0.0413	0.0386	23.9%	28.91%

Table 16 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on sprung mass displacement response

Parameter	RMS-passive	RMS-PID controller	RMS-LQR controller	(KPI_{RMS}) PID	(KPI_{RMS}) LQR
Sprung mass displacement (zs) (m)	0.0161	0.0113	0.0104	29.8%	35.4%

Table15 shows the average un-sprung mass displacement reduction for an active using (LQR and PID controllers) and Passive suspension system. From the table, it appears that the LQR and PID method results 23.9% and 28.91% reduction of the overshoot respectively. Compared to PID and LQR has good performance in reduction of the overshoot caused by the two-bump road profile in the positive peak direction. Comparing the two-suspension system, PID and LQR controlled active suspension has small oscillation in the reduction to sprung mass displacement than that of the passive suspension system as shown in table 16.

4.2.5 Sprung mass velocity for active and Passive suspension system

Figure 4.6 shows an active suspension system (using LQR and PID) compared to a passive suspension system.

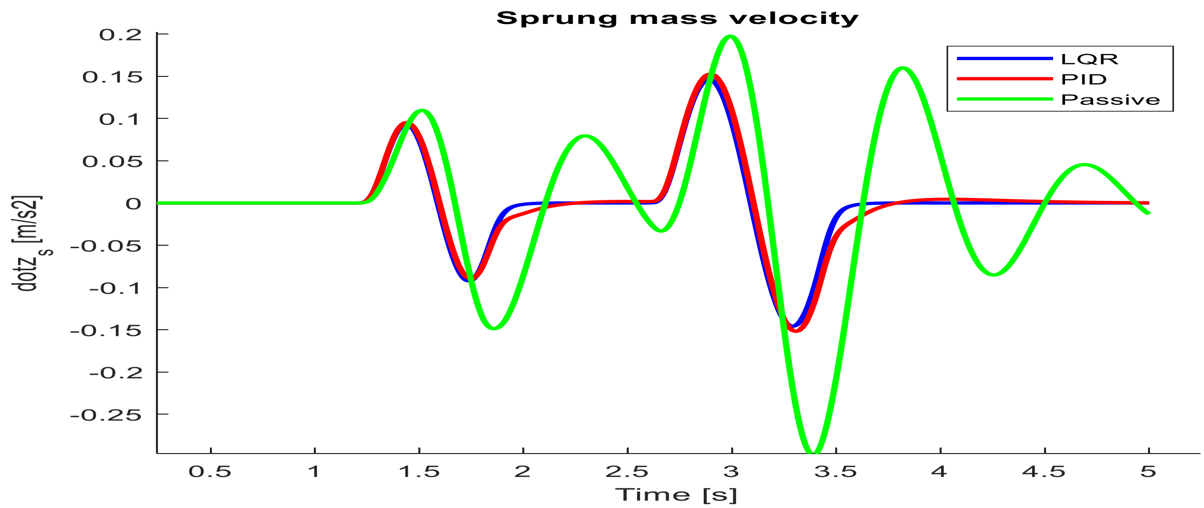

Figure 4.6 sprung mass velocity for passive and active (with PID and LQR) controllers

the PID and LQR controllers have minimum overshoot and have a good settling time than the conventional system.

Table 17 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on Sprung mass velocity response

Parameter	MAX - passive	MAX- PID controller	MAX- LQR controller	(KPI_{MAX}) PID	(KPI_{MAX}) LQR
Sprung mass velocity (($\dot{z}s$) (m/s))	0.1973	0.1524	0.1461	22.7%	25.6%

Table 18 KPI_{RMS} for ASS (PID and LQR controllers) and PSS on Sprung mass velocity response

Parameter	RMS-passive	RMS-PID controller	RMS-LQR controller	(KPI_{RMS}) PID	(KPI_{RMS}) LQR
Sprung mass velocity (($\dot{z}s$) (m/s))	0.0924	0.0500	0.0475	45.8%	48.5%

In reducing the overshoot of the system both LQR and PID controllers performed great but comparing PID and LQR controllers LQR has greater reduction in overshoot of the system which is 2.9% greater than that of the PID controlled active suspension system. Comparing the two-suspension system, PID and LQR controlled active suspension has small oscillation on sprung mass velocity than that of the passive suspension system and the RMS value of the LQR is 0.0475m/s which less compared to PID which shows as the LQR control strategy for the system has highest reduction to oscillation which result in fastest settling time as shown in table 18.

4.2.6 Sprung mass acceleration for active and Passive suspension system

Figure 4.7 shows the two suspension system suspension systems experience periods of positive and negative acceleration, which means the body is moving up and down. However, the magnitude of the acceleration is significantly lower for the LQR and PID controlled active suspension system compared to the Passive suspension systems. Both LQR and PID controlled active suspension system have greater reduction in overall acceleration and having the fastest settling time. Their curve reached the zero-acceleration band and stay within it relatively quickly compared to the others. The PID system has a higher acceleration reduction compared the LQR controller. The Passive suspension system has the highest acceleration of the three and has the slowest settling time. Its oscillations might take longer to die down and reach the zero-acceleration band as shown in the figure below.

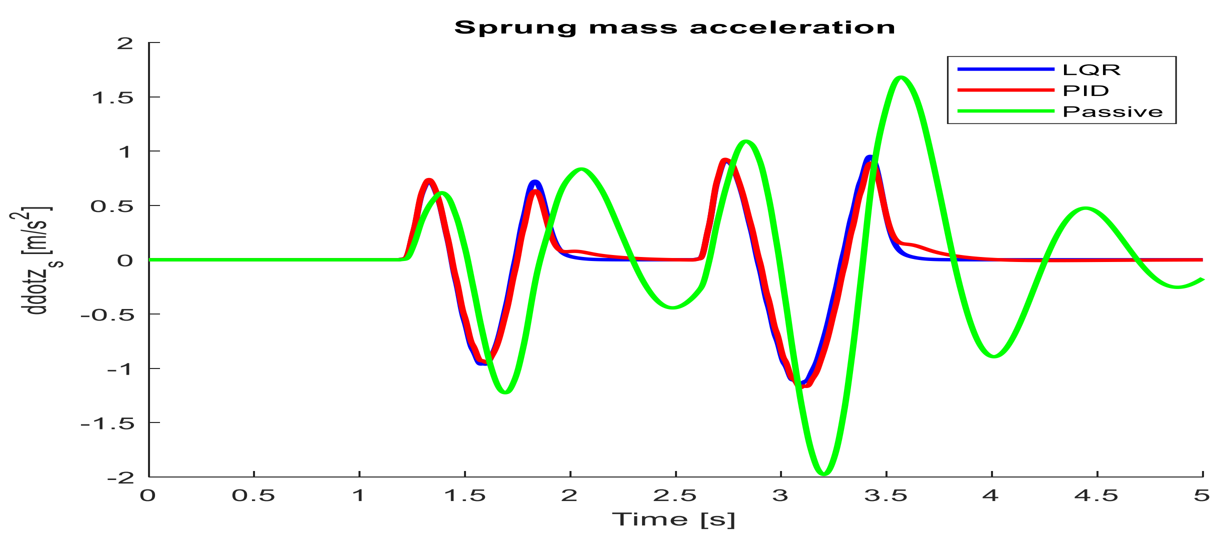

Figure 4.7 Sprung mass acceleration for passive and active (with PID and LQR) controllers

Table 19 shows the key performance index analysis of PID and LQR controllers in reduction of the sprung mass acceleration from peak to peak which is compared to the acceleration of the passive suspension system.

Table 19 KPI_{MAX} for ASS (PID and LQR controllers) and PSS on Sprung mass acceleration response

Parameter	MAX - passive	MAX- PID controller	MAX- LQR controller	(KPI_{MAX}) PID	(KPI_{MAX}) LQR
Sprung mass acceleration ($\ddot{z}s$) m/s^2	1.6811	0.9243	0.9519	45.07%	43.3%

Table 20 KPI_{RMS} for ASS (PID and LQR controllers) and PSS on Sprung mass acceleration response

Parameter	RMS-passive	RMS-PID controller	RMS-LQR controller	(KPI_{RMS}) PID	(KPI_{RMS}) LQR
Sprung mass acceleration ($\ddot{z}s$) m/s^2	0.6760	0.3730	0.3743	44.7%	44.58%

Table 19 and 20 shows that the LQR and PID control systems are the most effective at reducing the sprung mass acceleration of the suspension system, which would likely lead to a smoother ride. The PID system has reduced 45.07% overshoot to the positive peak direction and 44.7% reduction of oscillation. LQR has reduced 43.3% overshoot to the positive peak direction and 44.8% reduction of oscillation. The passive system results in the most significant sprung mass acceleration, which would likely result in a rougher ride.

4.2.7 Actuator force generated for active suspension system

Figure 4.8 shows two curves that shows the result of two different control strategies, LQR and PID for an active suspension system.

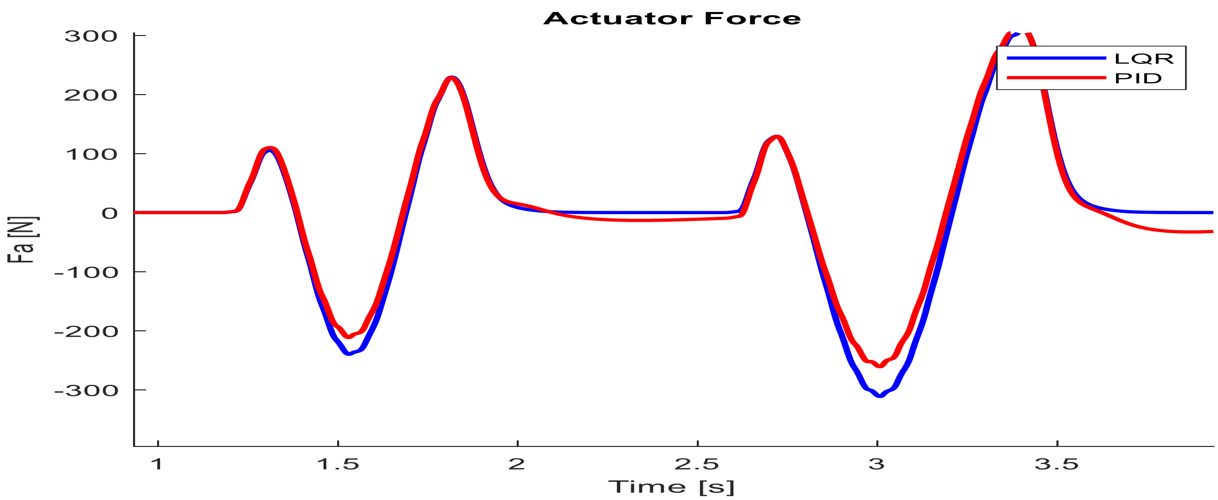

Figure 4.8 Actuator force generated for two bump road profile

Figure 4.8 shows that the generated force by the PID gains and the LQR gain to control the system sprung mass acceleration, the suspension travel and all the key performance index parameters discussed above, the table below shows the generated force for an active suspension system using PID and LQR controllers.

Table 21 Generated force for active suspension system using PID and LQR controllers to negative and positive peak

Parameter	MAX PID controller	MAX LQR controller	MIN for PID	MIN for LQR
Actuator force (Fa) (N)	311.5832	309.3595	-260.8787	311.2175

From table 21 the generated force using PID controller from the positive peak direction is 311.58N this amount of force is used to have good road ride comfort and road handling caused by the two-bump road profile. and by using LQR, 309.35N force from the positive peak direction is generated by LQR to minimize the overshoot and oscillations in of the sprung mass and un-sprung mass of suspension system.

4.3 Performance analysis of PSS and ASS with PSO-PID, FUZZY-PSO and LQR with two Bump road input profile

In this analysis two variables that can represent ride comfort and road handling on quarter car active and passive suspension system are selected for the comparison the controller used for the active suspension system are PSO-PID, FUZZY-PSO and LQR and additionally other state variables are added for comparison in order to assess the performance of the proposed controllers. and the response of the system is tested considering a speed bump profile, which corresponds to Figure 4.1.

4.3.1 Parameter tuning of PSO – PID, FUZZY-PSO and LQR controllers

The Q and R matrix that are presented on equation 4.1 are used as a parameter whereas for PSO-PID and FUZZY-PSO controllers the tuned parameters used are presented in the table 22. and by using equation (4.3 and 4.4) the percentage reduction of oscillation and overshot is discussed below

Table 22 PSO tuned result for PID 1 controller parameters

Control variable	Kp_2	Ki_2	Kd_2
$\ddot{Z}_s$	196.8	367.07	1768.3

Table 23 PSO tuned result for PID 2 controller parameters

Control variable	Kp	Ki	Kd
zs_zus	381.3	166.16	4000

Table 24 PSO tuned result for FUZZY controller parameters

Control variables	P	S	O
zs, $\dot{z}s$ *and* Fa	9.5768	2.491	1955.6

4.3.2 Suspension travel response of PSS and ASS

Figure 4.9 shows both suspension systems compress to lessen the impact. When the response of the controllers reached their highest value at the bumps peak, signifying the suspension's most compressed state. The suspension will stretch (rebound) to bring the vehicle back to its regular ride height after it has passed over the bump.

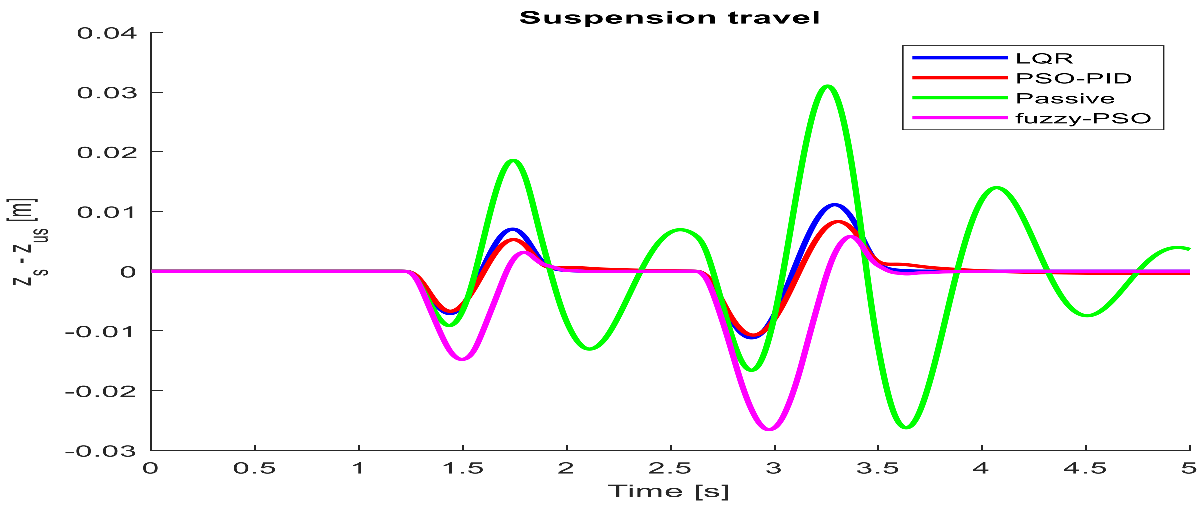

Figure 4.9 Suspension travel for PSS and ASS with PSO-PID, FUZZY-PSO and LQR with one Bump road input signal

Peak values of the LQR, PSO-PID, and FUZZY-PSO controlled active suspension system are compared from peak to peak. The suspension travel response of PSS and ASS with PSO-PID, FUZZY-PSO and LQR with two Bump road input signal is analyzed in reduction to overshoot and reduction to oscillation in the table 25 and table 26.

Table 25 KPI_{MAX} for ASS (PSO- PID, FUZZY-PSO and LQR controller) and PSS of suspension travel response

Parameter	MAX passive	MAX PSO-PID controller	MAX LQR controller	MAX FUZZY-PSO controller	(KPI_{MAX}) PSO-PID	$KPI_{MAX})$ FUZZY-PSO	(KPI_{MAX}) LQR
Suspension travel $(zs - zus)$ (m)	0.0310	0.0083	0.0111	0.0058	73.22%	81.29%	64.19%

The peak overshoot for the conventional passive suspension system suspension travel with the two-bump road input has a maximum value of 0.0310 m which is very high compared to the suspension travel peak overshoot of the active suspension system using PSO-PID, FUZZY-PSO and LQR controllers. Comparing the proposed controllers PSO tuned fuzzy controller shows a highest reduction in overshoot of the system followed by PSO-PID and LQR controllers respectively. The table below shows the percentage reduction of oscillation of suspension travel with the two-bump road profile.

Table 26 KPI_{RMS} for ASS (PSO- PID, FUZZY-PSO and LQR controller) and PSS of suspension travel response

Parameter	RMS passive	RMS PSO-PID controller	RMS LQR controller	RMS FUZZY-PSO controller	(KPI_{RMS}) PSO-PID	(KPI_{RMS}) FUZZY-PSO	(KPI_{RMS}) LQR
Suspension travel $(zs - zus)$ (m)	0.0105	0.0032	0.0036	0.0029	69.5%	72.3%	65.7%

From the table, it appears that the root mean square value for the passive suspension system is greater compared to the active suspension system with the given controllers. The fuzzy-PSO method results in the lowest percentage reduction of oscillation the system by 72.3% compared to PSO-PID and LQR.

4.3.3 Sprung mass displacement response of PSS and ASS with two Bump road input signal

Figure 4.10 shows the vertical displacement of the PSS and ASS with the given road input signal

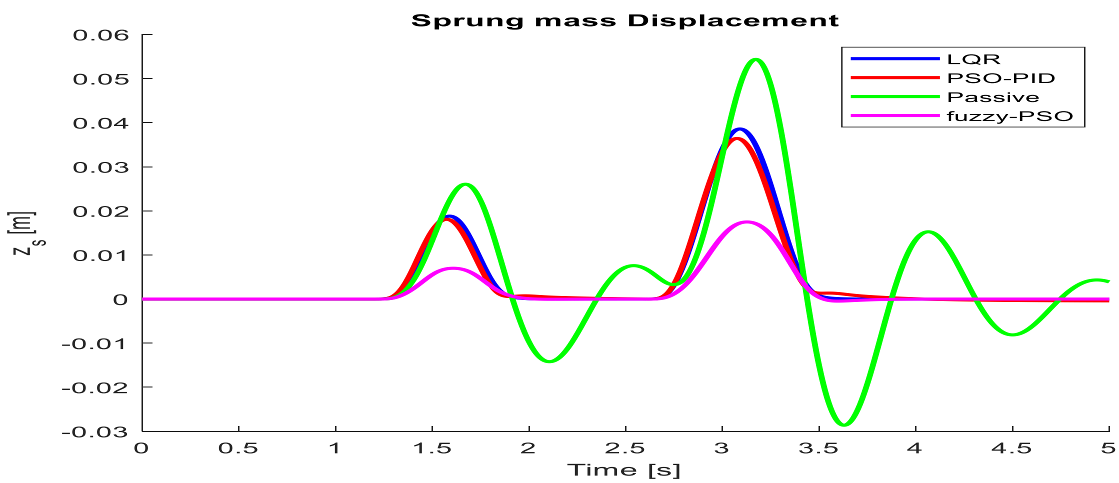

Figure 4.10 sprung mass displacement for PSS and ASS with PSO-PID, FUZZY-PSO and LQR with two Bump road input signal

The magnitude of the displacement is significantly lower for the PSO-PID, FUZZY-PID and LQR controlled ASS compared to the PSS. The peak-to-peak value comparison of the PSO-PID, fuzzy-PID and LQR controllers reduces the sprung mass displacement to the positive peak direction as well as to the negative peak direction. Table 27 and 28 describes the percentage reduction of overshoot and oscillation of the systems sprung mass velocity by using the proposed controllers.

Table 27 $KPI_{MAx}$for ASS (PSO- PID, FUZZY-PSO and LQR controller) and PSS of sprung mass displacement response

Parameter	MAX passive	MAX PSO-PID controller	MAX LQR controller	MAX FUZZY-PSO controller	(KPI_{MAx}) PSO-PID	(KPI_{MAx}) Fuzzy-PSO	(KPI_{MAx}) LQR
Sprung mass displacement (zs) (m)	0.0543	0.0364	0.0386	0.0175	36.2%	67.77%	28.9%

From the above table the passive suspension system has the highest peak overshot than the active suspension system. An active suspension system with fuzzy-PSO shows a highest reduction overshoot followed by PSO-PID and LQR controllers.

Table 28 $KPI_{RMS}$for ASS (PSO- PID, FUZZY-PSO and LQR controller) and PSS on sprung mass displacement response

Parameter	rms passive	rms PSO-PID controller	rms LQR controller	rms FUZZY-PSO controller	(KPI rms) PSO-PID	KPI rms) Fuzzy-PSO	(KPI rms) LQR
Sprung mass displacement (zs) (m))	0.0161	0.0098	0.0104	0.0048	39.13%	70.1%	35.4%

From the table 28 the passive suspension system has the highest peak overshoot than the active suspension system. The system with fuzzy-PSO shows a highest reduction overshoot on sprung mass displacement followed by PSO-PID and LQR controllers.

4.3.4 Sprung mass velocity response of PSS and ASS

A comparison between an ASS (FUZZY-PSO, LQR, or PSO-PID) and a passive suspension system is shown in Figure 4.11. For passive and active suspension systems with PSO-PID, FUZZY-PSO, and LQR, the four curves show the sprung mass velocity. The graph indicates that there are times of positive and negative velocity for both suspension systems, indicating that the sprung mass is rising and falling.

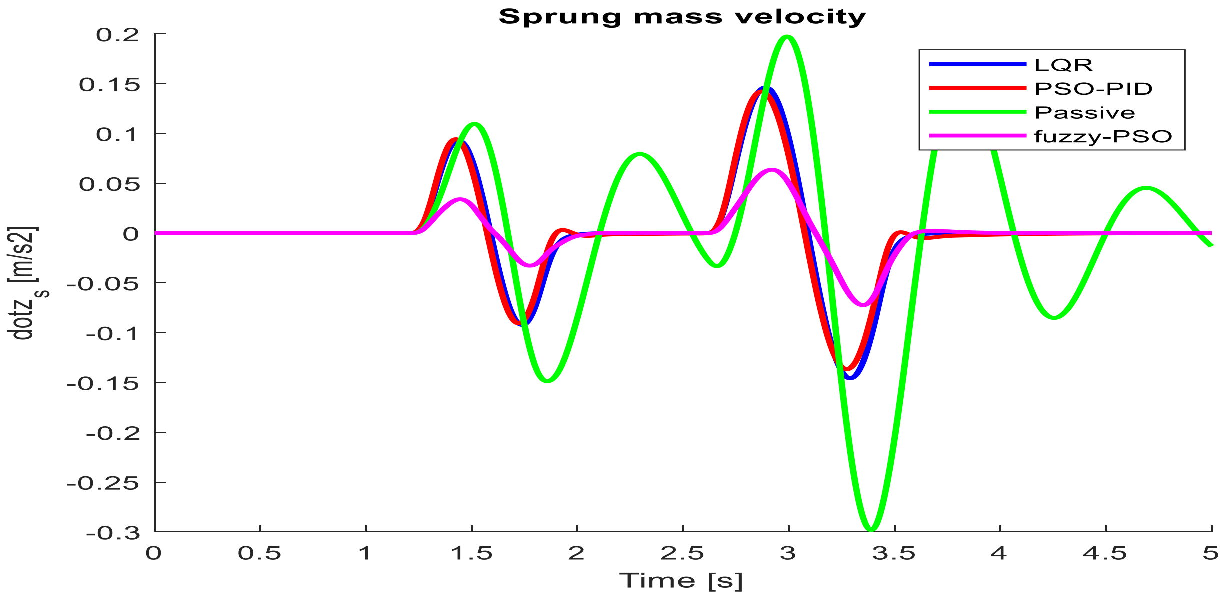

Figure 4.11 sprung mass velocity for PSS and ASS with PSO-PID, FUZZY-PSO and LQR with two Bump road input signal

However, The FUZZY-PSO controlled active suspension system has the lowest overall velocity of minimum value for both positive and negative peak direction followed by the PSO-PID and LQR controllers and the conventional passive suspension system.

Table 29 KPI_{MAX} for ASS (PSO- PID, FUZZY-PSO and LQR controllers) and PSS of sprung mass velocity response

Parameter	MAX passive	MAX PSO-PID controller	MAX LQR controller	MAX FUZZY-PSO controller	(KPI_{MAx}) PSO-PID	(KPI_{MAx}) Fuzzy-PSO	(KPI_{MAx}) LQR
Sprung mass	0.1973	0.1421	0.1461	0.0636	28%	67.76%	25.6%

velocity (zs) (m)							

From table 29 the magnitude of the velocity is significantly lower for the active suspension with fuzzy-PSO control system compared to the Passive and ASS with PSO-PID, LQR control system. Which means with the given road input the active suspension system using fuzzy-PSO has highest percentage reduction in overshoot of the system than the PSO-PID and LQR controllers. Table 30 shows the root mean squared value for passive and active suspension system with respect to reduction of overshoot and oscillation.

Table 30 $KPI_{RMS}$for ASS (PSO- PID, FUZZY-PSO and LQR controllers) and PSS of sprung mass velocity response

Parameter	RMS passive	RMS PSO-PID controller	RMS LQR controller	RMS FUZZY-PSO controller	(KPI_{RMS}) PSO-PID	(KPI_{RMS}) Fuzzy-PSO	(KPI_{RMS}) LQR
Sprung mass acceleration (zs) (m)	0.0924	0.0453	0.0475	0.0207	50.9%	77.6%	48.6%

The table describes the system with fuzzy-PSO has highest percentage in reduction of the oscillation compared to PSO-PID and LQR controllers. Which shows fuzzy-PSO has the fastest settling time which reached the zero-velocity band and stay within it relatively quickly compared to the others.

4.3.5 Sprung mass acceleration response of PSS and ASS

The figure 4.12 shows the sprung mass acceleration of a suspension system over time. The four curves represent the acceleration of the sprung mass for passive and active suspension systems with PSO-PID, FUZZY-PSO and LQR. The graph shows that both suspension systems experience periods of positive and negative acceleration, which means the sprung mass is moving up and down.

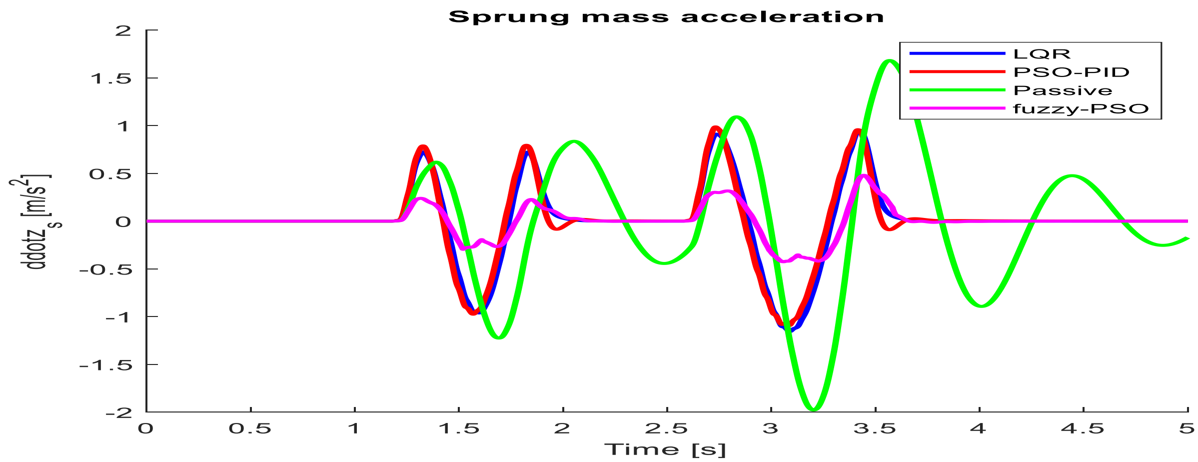

Figure 4.12 sprung mass acceleration for passive and active suspension system using fuzzy-PSO, PSO-PID and LQR controllers

When comparing the passive and active suspension system with PSO-PID, fuzzy-PSO and LQR control system, the active suspension systems acceleration is noticeably smaller. the fuzzy-PSO system has the lowest total acceleration and has the quickest rate of settling. Compared to the others, its curve entered the zero-acceleration region and stayed there rather rapidly. But the passive system results in discomfort during ride.

Table 31 KPI_{MAX} for ASS (PSO- PID, FUZZY-PSO and LQR controllers) and PSS of sprung mass acceleration response

Parameter	MAX passive	MAX PSO-PID controller	MAX LQR controller	MAX FUZZY-PSO controller	(KPI_{MAX}) PSO-PID	(KPI_{MAX}) Fuzzy-PSO	(KPI_{MAX}) LQR
Sprung mass acceleration (zs) (m)	1.6811	0.8766	0.9519	0.4795	47.85%	71.4%	43.3%

Table 31 shows the key performance index analysis of fuzzy-PSO, PSO-PID and LQR controllers in reduction of the sprung mass acceleration from peak to peak which is compared to the sprung mass acceleration of the passive suspension system. from the result fuzzy-PSO has highest percentage of reduction compared to PSO-PID and LQR controllers. The root mean square value of passive and active suspension system is compared and presented in the table below.

Table 32 $KPI_{RMS}$for ASS (PSO- PID, FUZZY-PSO and LQR controllers) and PSS of sprung mass acceleration response

Parameter	RMS passiv e	RMS PSO-PID controlle r	RMS LQR controller	RMS FUZZY-PSO controller	(KPI_{RMS}) PSO-PID	(KPI_{RMS}) Fuzzy-PSO	(KPI_{RMS}) LQR
Sprung mass acceleratio n (zs) (m)	0.6760	0.3724	0.3743	0.1534	45%	77.3%	44.6%

4.4 Actuator force generated for active suspension system using PSO-PID, FUZZY-PSO and LQR controllers

Figure 4.13 shows the result of three different control strategies, fuzzy-PSO, LQR and PSO- PID for an active suspension system.

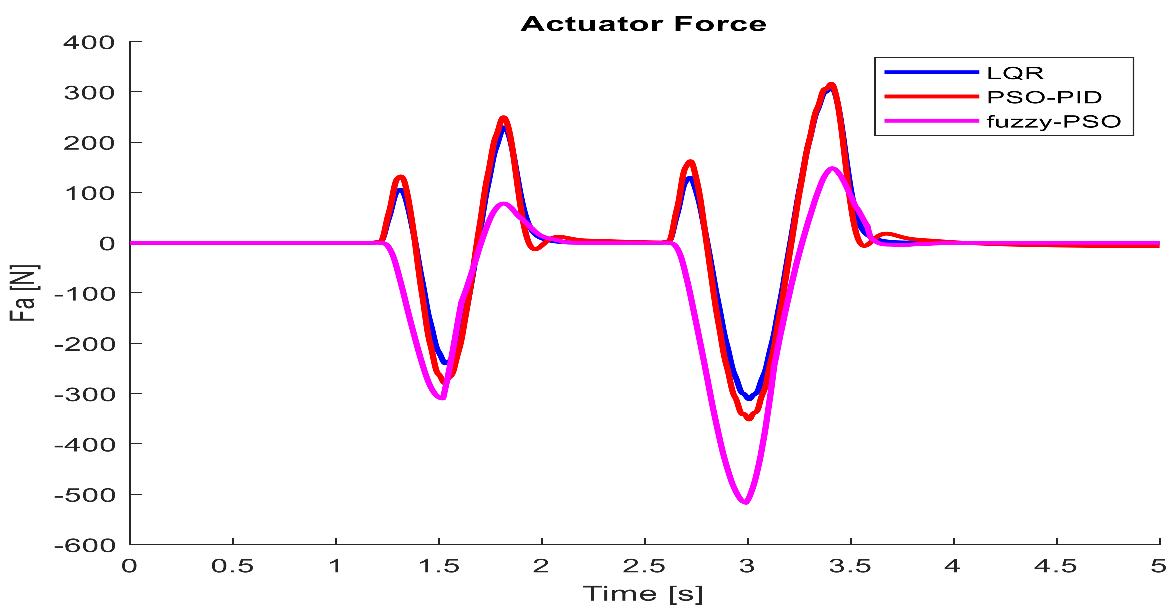

Figure 4.13 generated actuator force by fuzzy-PSO, PSO-PID and LQR controller

Table 33 shows the maximum and the minimum value of the actuator force generated using the three controllers.

Table 33 generated actuator force for active suspension system using fuzzy-PSO, PSO-PID and LQR controllers

Parameter	Max PSO_PID controller	Max LQR controller	Max(fuzzy-PSO)	Min for PID	Min for LQR	Min (PSO-fuzzy)
Actuator force (Fa) (N)	315.9754	309.3595	148	351.3914	311.2175	517.0724

From table 33 the generated force using fuzzy-PSO controller from the positive peak direction is 148N and this amount of force is used to have good road ride comfort and road handling caused by the two-bump road profile. and by using PSO-PID and LQR, 315.9754N and 309.3595N force from the positive peak direction is generated respectively to minimize the overshoot and oscillations of the sprung mass and un-sprung mass of suspension system. To the negative peak direction fuzzy-PSO has generated higher force to have stable system compared to PSO-PID and LQR controllers.

CHAPTER FIVE

5. CONCLUSION AND RECOMMENDATION

5.1 Conclusion

PID, LQR, PSO_PID, FUZZY_PSO control techniques have been effectively executed in this thesis work on a quarter car active suspension system model. The objective of minimizing the road with bump disturbances of a quarter car model is achieved successfully using controllers PID, LQR, PSO_PID, FUZZY_PSO and shows that in an active suspension system with an actuator force can successfully minimize the vibration from uneven road input. The purpose of using controllers in active suspension system is to improve ride comfort as well as road handling during different driving conditions on rough road surface. So, for this research paper two parameters are used for comparison, the first one is suspension travel and sprung mass acceleration. The response of suspension travel with two bump road surface for an active suspension system using LQR and PID controllers is decreased the overshoot by 64.2% and 51.6% respectively which compared to passive suspension system. Therefore, by using LQR controller we can minimize the overshoot of the conventional suspension system by more than half percent which shows great reduction in overshoot compared to the active suspension system controlled by PID controller. In reduction to oscillation and settling time of the system, LQR and PID controlled active suspension system Suspension travel has a value of 65.7% and 58.09% respectively this shows as the oscillation is decreased by 58.09% using PID and 65.9% by LQR. So that the designed LQR controllers for an active suspension system has good road handling capacity followed by PID controller both are compared with the passive suspension system. In case of minimizing the body acceleration both PID and LQR controlled active suspension system has decreased the overshoot of the conventional system by 45.07% and 43.05% respectively so using PID controller for an active suspension system can lead to good ride comfort compared to LQR and passive suspension system. In reduction to the oscillation of the sprung mass root mean square value of LQR and PID is 44.7 percent and 44.58 percent, this shows both PID and LQR controllers has decreased the oscillation when compared to the passive suspension system. Finally, properly selecting the gains of PID controller and the weighting factors (Q and R) matrices for LQR controller can result in good road handling and ride comfort for the passengers by generating an actuator force that can be used to minimize the body acceleration and suspension travel oscillation as well as maximum overshoot.

To overcome the error during setting initial values for the two PID gains, an additional study is done to find the optimal value of the PID gains (KP_2, Ki_2, Kd_2, Kp ,Ki and Kd and scaling factors(P, S and O) for the two inputs and output of the fuzzy logic control system. A particle swarm optimization technique is used to find the optimum value of the PID gains and the scaling factor for the fuzzy logic controller. As the PID and LQR here also the maximum key performance index (reduction of overshoot) and the root mean square values (reduction of oscillation) of the suspension travel and the sprung mass acceleration of an active suspension system is compared to the passive suspension system. So that the PSO tuned fuzzy logic controller has a highest reduction in the overshoot of the sprung mass acceleration of the system with two bump road input by 71.4% followed by PSO-PID and LQR controlled active suspension system and in reduction to oscillation also the FUZZY-PSO controlled active suspension system has highest reduction by 77.3% compared to PSO-PID controlled active suspension system so we can conclude that both PSO-PID and FUZZY PSO controller can give ride comfort. but comparing the conventional PID with PSO tuned PID the PSO tuned PID has a highest reduction in oscillation as well as overshoot. And comparing the response of suspension travel FUZZY-PSO has 81.29% overshoot reduction and 72.3% reduction in oscillation compared to the passive suspension system. Therefore, by using parameter tuning techniques it is possible to increase the performance of controller in this case both PSO-PID and FUZZY-PSO have good performance to have good ride comfort and road handling compared to conventional PID and LQR controller for an active suspension system.

5.2 Recommendation

Under several road profiles and vehicle speed procedures, an active suspension system with the controller is effective for a forming dynamic model for improving ride comfort and road handling. It also suggests that by using the half car and full car models of the suspension system the response can be different using the complex model of a system the yaw and the rolling response of the system should have to be analyzed for better result. The author suggests that interested researchers can design and tune PID and FUZZY logic controllers instead of PSO by other tuning techniques like genetic algorithm, ANFIS and so on for a complex vehicle model to improve the vehicle performance on numerous road profiles, including those with un-even road profiles.

5.3 Future work

The aim of this study is to use a quarter car model and to forecast the performance of an active suspension system. This is a condensed version of a comprehensive automotive model that ignores some characteristics of the vehicle's behavior, such rolling and yaw. To get the best outcomes in these domains as well, a more comprehensive model that accounts for various degrees of freedom has to be researched. In order to conduct a thorough vehicle dynamic analysis that incorporates a non-linear system to enhance vehicle driving under varied road profiles and vehicle speeds, the researchers intend to build the model into an entire automobile. Additionally, future studies must consider actual implementation.

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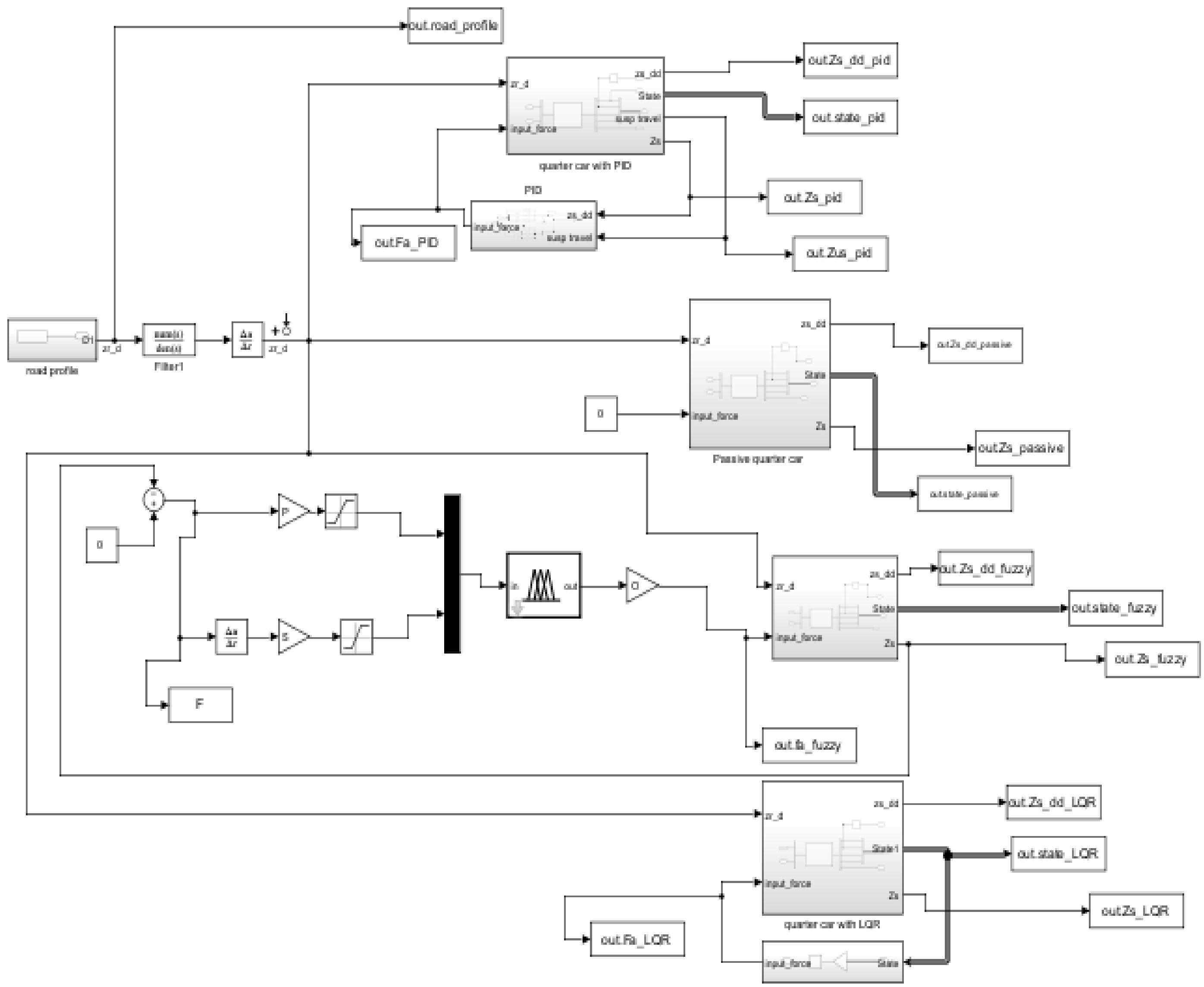
APPENDIXES

Appendix A: Simulink model for ASS with PID, PSO-PID, FUZZY-PSO and LQR and PSS



Appendix B: Road Profile MATLAB Code

```
% % one bump Road profile
T_f = 5;
sim_time = 0.01:0.04:T_f;
road_surface = zeros(2,length(sim_time));

% choose velocity
road = 1; % 20 km/hr
 road = 2; % 40 km/hr
```

```
road = 3; % 72 km/hr

for i = 1:length(sim_time)
    road_surface(1,i) = sim_time(i);
    if ((sim_time(i)>1) && (sim_time(i)<1.09)) && road == 1 % 20 km/hr
        road_surface(2,i) = (0.04*(1-cos(7*pi*sim_time(i))));
    elseif ((sim_time(i)>1) && (sim_time(i)<1.045)) && road == 2 % 40
km/hr
        road_surface(2,i) = (0.04*(1-cos(7*pi*sim_time(i))));
    elseif ((sim_time(i)>1) && (sim_time(i)<1.025)) && road == 3 % 72
km/hr
        road_surface(2,i) = (0.04*(1-cos(7*pi*sim_time(i))));
    else
        road_surface(2,i) = 0;
    end
%
End


% Two bump road profile

% Define bump properties
bump_height1 = 0.02;
bump_height2 = 0.04;
bump_width1 = 0.6; % Wider bumps for smoother transition
bump_width2 = 0.8;

% Define bump centers
bump1_center = 1.5;
bump2_center = 3.0;

% Define scenario (1: One bump, 2: Two bumps)
scenario = 2;

for i = 1:length(sim_time)
    road_surface(1, i) = sim_time(i);

    % Define road profile based on scenario
    if scenario == 1
        if (sim_time(i) >= bump1_center - bump_width1 / 2) &&
(sim_time(i) <= bump1_center + bump_width1 / 2)
            road_surface(2, i) = bump_height1 * (1 - cos(2 * pi *
(sim_time(i) - (bump1_center - bump_width1 / 2)) / bump_width1)) / 2;
        else
            road_surface(2, i) = 0;  % Flat road
        end
```

```
    elseif scenario == 2
        if (sim_time(i) >= bump1_center - bump_width1 / 2) &&
(sim_time(i) <= bump1_center + bump_width1 / 2)
            road_surface(2, i) = bump_height1 * (1 - cos(2 * pi *
(sim_time(i) - (bump1_center - bump_width1 / 2)) / bump_width1)) / 2;
        elseif (sim_time(i) >= bump2_center - bump_width2 / 2) &&
(sim_time(i) <= bump2_center + bump_width2 / 2)
            road_surface(2, i) = bump_height2 * (1 - cos(2 * pi *
(sim_time(i) - (bump2_center - bump_width2 / 2)) / bump_width2)) / 2;
        else
            road_surface(2, i) = 0;  % Flat road
        end
    else
        error('Invalid scenario. Choose 1 or 2.');
    end
end

road_surface = timeseries(road_surface(2,:),sim_time);
```

Appendix C: Particle swarm optimization tuned FUZZY and PID controller MATLAB code

```
% pso main program start %
max_iterations=10 ;    % set maximum number of iteration

max_no_runs=1;      % set maximum number of runs need to be
for run=1:max_no_runs
    run;
    % pso initialization start %
    for i=1:n
        for j=1:m
            x0(i,j)=round(LowerLimit(j)+rand()*(UpperLimit(j)-
LowerLimit(j)));
        end
    end
    x=x0;       % initial population %
    for i=1:n
        for j=1:m
            v(i,j)=(round(LowerLimit(j)+rand()*(UpperLimit(j)-
LowerLimit(j))))*.1;
        end
    end

    for i=1:n

        Kp_2 = x(i,1);
        Ki_2 = x(i,2) ;
```

```
        Kd_2 = x(i,3) ;
        Kp = x(i,4);
        Ki =x(i,5);
        Kd =x(i,6);
        P = x(i,7);
        S = x(i,8);
        O = x(i,9);
        sim('qcar.slx');
        load('F2.mat')
        f0(i,1) = obj(end);

        load('F.mat')
        f01(i,1) = obj_f(end);


    end
    [fmin0,index0]=min(f0);
    [fmin01,index01]=min(f01);
    ParticleBest=x0;
    ParticleBest1=x0;% initial ParticleBest
    GlobalBest=x0(index0,:);      % initial GlobalBest
    GlobalBest1=x0(index01,:);
    % pso initialization end %

    % pso algorithm start %
    Iteration=1;
    while Iteration<=max_iterations

        % pso velocity updates %
        for i=1:n
            for j=1:m
                v(i,j)=w*v(i,j)+c1*rand()*(ParticleBest(i,j)-
x(i,j))...
                        +c2*rand()*(GlobalBest(1,j)-x(i,j));
                v(i,j)=w*v(i,j)+c1*rand()*(ParticleBest(i,j)-
x(i,j))...
                        +c2*rand()*(GlobalBest1(1,j)-x(i,j));
            end
        end

        % pso position update %
        for i=1:n
            for j=1:m
                x(i,j)=x(i,j)+v(i,j);
            end
        end
```

```
% handling boundary violations %
for i=1:n
    for j=1:m
        if x(i,j)<LowerLimit(j)
            x(i,j)=LowerLimit(j);
        elseif x(i,j)>UpperLimit(j)
            x(i,j)=UpperLimit(j);
        end
    end
end

% evaluating fitness %
for i=1:n

   Kp_2 = x(i,1);
Ki_2 = x(i,2) ;
Kd_2 = x(i,3) ;
Kp = x(i,4);
Ki =x(i,5);
Kd =x(i,6);
P = x(i,7);
S = x(i,8);
O = x(i,9);
sim('qcar.slx');
f(i,1) = obj(end);
f1(i,1) = obj_f(end);


end

% updating ParticleBest and fitness %
for i=1:n
    if f(i,1)<f0(i,1) && f1(i,1) < f01(i,1)
        ParticleBest(i,:)=x(i,:);
        f0(i,1)=f(i,1);
        f01(i,1)=f1(i,1);
    end

end

[fmin,index]=min(f0);
[fmin1,index1]=min(f01);% finding out the best particle
ffmin(Iteration,run)=fmin;
ffmin(Iteration,run)=fmin1; % storing best fitness
ffite(run)=Iteration;         % storing iteration count
```

```
        % updating GlobalBest and best fitness %
        if fmin<fmin0 && fmin1<fmin01
            GlobalBest=ParticleBest(index,:);
            GlobalBest1=ParticleBest(index1,:);

            fmin0=fmin;
            fmin01 = fmin1;
        end


        % displaying iterative results %
        if Iteration==1
            fprintf('Iteration    Best particle    Objective
function\n');
        end
        fprintf('%8g  %8g          %8.4f\n',Iteration,index,fmin0);
        Iteration=Iteration+1;
    end
    % pso algorithm %
    GlobalBest;
    GlobalBest1;
    fff(run)=fmin0;
    fff(run)=fmin01;

    rgbest(run,:)=GlobalBest;

        Kp_2 ;
        Ki_2 ;
        Kd_2 ;
        Kp;
        Ki;
        Kd;
    rgbest(run,:)=GlobalBest1;
        P;
        S;
        O;
end
% pso main program end %
disp(newline);

[BestFunction,bestrun]=min(fff)
best_variables=rgbest(bestrun,:)
```